Overview
This text has been written to help students who have struggled with previous more traditional instruction of prealgebra mathematics through elementary algebra. Using a non-traditional approach free of gimmicks and rote memorization, students will experience math more like language learners and less like calculators. Examples are written with process based explanations providing consistent and coherent connections supporting students of multiple levels of mathematic experiences.

Composition of Mathematics, Algebra Transitions:
- Replaces the traditional order of operations (PEMDAS) and provides students a common structure (The Composition of Mathematics) to evaluate arithmetic and work with algebra.
- Enables students to more intuitively navigate the transition from pre-algebra mathematics to algebra content.
- Teaches students to “read” and translate mathematics by identifying terms and factors, which in turn defines process of evaluating or simplifying expressions and solving equations.
- Empowers students to navigate more complicated mathematics by developing habits of effective mathematicians including; identifying composition prior to problem solving, taking advantage of properties of real numbers, and developing a clear step-by-step problem structure.
- Provides students with a more intuitive and coherent algebra foundation which they can build on in subsequent mathematics courses including; Intermediate Algebra, Liberal Arts Math and/or Introductory Statistics.

Introduction
Algebra is a universal language used to explain quantitative relationships and define mathematical procedures or process performed on one or more quantities. In order to understand algebra you need to learn it in much the same way you would learn a new language. To read and understand an English sentence, you need to be able to identify and pronounce each word in the sentence. But, in order to pronounce a word, you would have to be able to identify and pronounce each letter in the word. These components of English and how they are related to each other can be referred to as the Composition of English (Figure 1). Mathematics and Algebra work in relatively the same way. In order to solve an algebraic equation or calculate the value of an expression, you would first need to identify and calculate the value of each term in the expression. But, to evaluate a term, you would need the ability to identify and calculate the value of each factor in the term. We will refer to these components of mathematics and how they are related to each other as the Composition of Mathematics.
Chapter 1
The Composition of Mathematics - Expressions

Just like letters form words and words form expressions and sentences in English, factors form terms and terms form expressions and equations in mathematics and algebra. In this chapter you will learn how to identify each of the components of mathematics that make up mathematical expressions. By understanding and utilizing the Composition of Mathematics (Figure 1.1), you will more intuitively understand order of operations, by evaluating complicated expressions term by term and factor by factor. You will also learn to use correct Problem Structure that will be the most efficient in keeping track of the components (terms and factors) when evaluating complicated multi-step expressions.

And, since algebra defines mathematical process, your transition to procedures and properties of algebraic expressions will also be based on this same Composition of Mathematics.

Since variables in algebra represent numeric values, then properties and rules of algebra must be defined by and follow all the properties and rules of mathematics. In this introduction we will revisit properties of real numbers and understand why we follow particular procedures in mathematics when we are adding or multiplying numbers together.

Adding in Mathematics:
When adding numbers with more than one digit, we write the addition in columns, lining up the digits from the right.

\[
\begin{array}{c}
1,324 \\
+ 451 \\
\hline
1,775
\end{array}
\]

Why? Because in mathematics, we can only add like things. So, we line up like digits and only add like digits together, keeping track of each digit as we go. The vertical format is used when adding numbers to help keep track of like digits. I will refer to the way in which step-by-step evaluations are written as the problem structure.

Note: The vertical format of addition is a good problem structure to help keep like digits together in columns.

Now consider this same example with each number written in expanded form. Notice in expanded form, we can recognize each like digit (ones, tens, hundreds and thousands) as a part of the expanded sum of terms. Also notice, we would not have to rewrite the numbers in columns in order to keep track of like digits to add them together.

\[
\begin{array}{c}
\frac{1,324}{1,000 + 300 + 20 + 4} \\
\frac{+ 451}{1,000 + 400 + 50 + 1} \\
\hline
\frac{1,775}{1,000 + 700 + 70 + 5}
\end{array}
\]

*To add the numbers, identify and combine like terms in the expanded form.

Note: The word term used here defines an addend of an expression. We will formally define term as it pertains to the Composition of Mathematics later in section 1.2.

You Try It!
Write the sum \(3,251 + 2,407\) in expanded form and add by combining like terms.
We are using the **Associative** and **Commutative Properties** of real numbers which allows us to rearrange and add numbers in any order. From this example we define a fundamental principle of mathematics.

**Fundamental Principle of Addition:** *Like terms* can be combined in any order.

We will take advantage of this fundamental principle by identifying and combining signed terms (*integers*) in any order in Section 1.1.

**Multiplying in Mathematics**

What is the procedure we use to multiply two numbers with more than one digit? Let’s go through an example and describe what we are doing at each step.

```
23 \times 12 \rightarrow \begin{array}{c}
23 \\
\times 12 \\
\hline
46 \\
+ 23 \\
\hline
276
\end{array}
```

Write vertically, lining up digits from the right.

The one’s digit, 2, is multiplied to every digit in the top number writing the answer starting in the one’s column.

The ten’s digit, 1, is multiplied through, but we move the answer one digit to the left starting in the ten’s column.

Finally, add the columns of like digits together.

To summarize, every digit from each number gets multiplied together using columns to keep track of like digits. We could call this method *digit-by-digit* multiplication.

Consider this multiplication with the numbers written in *expanded form*. Again, each digit in each number is represented by a *term* in each sum.

```
(20 + 5) \times (10 + 2) \\
\rightarrow 200 + 40 + 30 + 6 \\
\rightarrow 276
```

*To multiply, simply multiply *term-by-term* and then, add *like terms* together.*

Notice, we used the **Associative** and **Commutative Properties** of Real numbers, which allow us to rearrange and multiply numbers (*factors*) in any order. This process of multiplying *factors* is based on another fundamental principle of mathematics.

**Fundamental Principle of Multiplication:** *Factors* are multiplied *term-by-term* in any order.

We will first consider multiplying factors that involve only one term in Section 1.3 and then consider factors with more than one term (*grouped factors*) in Section 1.4.

These basic fundamental principles of mathematics must govern everything we do in mathematics. From evaluating arithmetic expressions to simplifying algebraic expressions and solving equations, every mathematical procedure we perform must be based on these basic fundamental principles of addition and multiplication. Therefore, addition and subtraction from left to right will be changed to combining *signed terms* in any order and multiplying and dividing from left to right will be changed to multiplying *fractional factors* in any order.

From these fundamental principles we will construct and define the **Composition of Mathematics**.

Discussion:

Why is “carrying” used when adding numbers together?

Add: 178 + 54

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You Try It!

Write the product $32 \times 21$ in *expanded form* and multiply term-by-term.

Multiply: $178 \times 54$

Discussion:

Why is “carrying” used when multiplying numbers together?

Multiply: $178 \times 54$
Section 1.1
Signed Terms

Now that we understand the fundamental principles of mathematics, we need to formally define all the components which make up the composition of mathematics. Utilizing the composition of mathematics, you will understand how the fundamental principles in mathematics transition to the basic fundamentals and principles of algebra.

Composition of Mathematics:
An expression is any combination of numbers, variables (letters) and operations.

Examples of Expressions:

a) \(3 - (-4) + 8\)  
b) \(-3x - 4(x + 3)\)  
c) \(-4\)  
d) \(\frac{2 - \frac{1}{3} - \frac{3}{5}}{\frac{11}{10}}\)  
e) \(\sqrt{-6 + 25 - 10}\) \(\frac{3^2 - 6}{3^2 - 6}\)  
f) \(2x^3 y\)

Objective 1  Identifying Signed Terms within an Expression

To calculate the value of an expression we first must be able to identify the value of every term in the expression.

Composition of Mathematics:
Expressions are made up of terms which are separated by + or -, outside of grouping symbols including ( ) or and all factor operators.

Example 1: Identify the separate terms in each expression by drawing a vertical dotted line between the terms. How many term(s) are in each expression? Do not evaluate.

a) \(-4 + 12 - 8\)  
Solution: \(-4 \mid +12 \mid -8\)  
3 terms

b) \(-3x - 4(x + 3)\)  
Solution: \(-3x \mid -4(x + 3)\)  
2 terms

c) \(-3(-4 + 8)\)  
Solution: \(-3 \mid -4 + 8\)  
1 term

d) \(7 - 3(-10) + 2 - (5 + 9)\)  
Solution: \(7 \mid -3(-10) \mid +2 \mid - (5 + 9)\)  
4 terms

Note: Factor operators will be defined and used later in section 1.2. We will only consider ( ) as grouping symbols in the expressions for this section.

Section Objectives
1. Identify Signed Terms within an Expression
2. Combining Signed Terms
   a) Combining Same Sign Terms
   b) Combining Opposite Signed Terms
3. Evaluating Expressions Containing Signed Terms
You Try It! Identify the separate terms in each expression by drawing a vertical dotted line between the terms. How many term(s) are in each expression? Do not evaluate.

a) \(12 - 5(x + 1) + 3x\)  
b) \(-3(-2) + 10 - 4(7 - 5) - 5\)

**Objective 2** Combining Signed Terms

Evaluating or simplifying an **expression** containing signed **terms** involves **combining integers** together. Remember, you can combine terms in any order.

**Integers** include all positive and negative whole numbers including 0. **Note**: 0 is neither positive nor negative. The integers are:

\[
\ldots \quad \ldots
\]

A number line can be used to represent the relative position of integers with respect to each other. Notice, 2 is 5 units to the right of -3.

Recall: By the **Fundamental Principle of Addition**, we are only allowed to **combine like terms** together in any order.

**Discussion**: By defining negative integers as moving left and positive integers as moving right, combining integers can be interpreted as a list of movements on the number line starting at zero.

Combine: \(-3 - 2\)

Where do you end up?

**Objective 2.a)** Combining Same Sign Terms

Same sign integers represent like terms. Therefore, we combine by adding the values and attaching the same sign.

**Example 2**: Write an expression that contains the following integer terms. Then, combine the terms.

a) Combine 5 and 7  
   **Expression**: \(5 + 7\) or \(7 + 5\)  
   **Answer**: 12

b) Combine \(-3\) and \(-2\)  
   **Expression**: \(-3 - 2\) or \(-2 - 3\)  
   **Answer**: \(-5\)

Remember, **like terms can be combined in any order**. As long as the expression involves the exact same terms, combining the terms will have the same value.

**Problem Structure Tip**: Practice writing answers below the expression (vertically) without an equals (=) sign. This will keep track of term values in more complicated problems.
Combining Opposite Signed Terms

Opposite signed integers represent opposite quantities and are not like terms. But, you can think of positive numbers as money in your pocket (credit) and negative numbers as money you owe (debt), then combining opposite integers is like finding your net worth.

To find net worth, find the difference between credit (+) and debt (-). If there is more credit (+) than debt (-), the net worth will be (+). If there is more debt (-) than credit (+), the net worth will be (-).

<table>
<thead>
<tr>
<th>Credit</th>
<th>Debt</th>
<th>Difference</th>
<th>+/- Net Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>+50</td>
<td>−20</td>
<td>50 − 20 = 30</td>
<td>+30</td>
</tr>
<tr>
<td>+24</td>
<td>−40</td>
<td>40 − 24 = 16</td>
<td>−16</td>
</tr>
<tr>
<td>0</td>
<td>−15</td>
<td>15 − 0 = 15</td>
<td>−15</td>
</tr>
</tbody>
</table>

Example 3: Write an expression that contains the following integer terms. Then, combine the terms.

**a)** Combine 12 and −9
Expression: 12 − 9 or −9 + 12
Answer: 3

**b)** Combine −8 and 5
Expression: −8 + 5 or 5 − 8
Answer: −3

Evaluating Expressions Involving Signed Terms

Evaluating an expression is the same as calculating the value of the expression.

By the Composition of Mathematics we know expressions are made up of terms, therefore, to evaluate or calculate the value of an expression you need to combine the value of all the terms in the expression.

**Composition of Mathematics:**
To evaluate or calculate the value of an expression:
1) Identify all the signed terms (integers) in the expression
2) Combine the term values together in any order.

The following examples show ways on how to take advantage of combining signed terms in any order. You can always combine terms left to right, but to improve your efficiency and accuracy when calculating, look for convenient terms to combine first.

It might be more efficient to group and combine all positive integers together and all negative integers together first, then combine the opposite signed integers together last.

Example 4: Evaluate the following expression.

\[
\begin{align*}
7 \quad \cdot -10 & \quad \cdot -5 \\
7 \quad \cdot -15 & \quad -8
\end{align*}
\]

Identify the separate signed terms (integers) in the expression.
*Use vertical lines to help keep track of separate signed terms as you work.

Combine all positive integers together and all negative integers together.

Combine the opposite signed integers together.
There is no reason to rewrite the terms in an expression in a different order. As long as an expression has the exact same terms, the value of the expression will be the same no matter what order the terms are written.

**Example 5:** Evaluate the following expressions.

a) \[13 - 9 - 6\]
   - Identify the separate signed terms (integers) in the expression.
   - Combine all positive integers together and all negative integers together.
   - Then, Combine the opposite signed integers together.

b) \[-6 + 13 - 9\]
   - Identify the separate signed terms (integers) in the expression.
   - Use vertical lines to help keep track of separate signed terms as you work.
   - Combine all positive integers together and all negative integers together.
   - Combine the opposite signed integers together.

Another way to take advantage of combining terms in any order is to look for **opposite integers** that combine to zero or opposite signed integers that are fairly easy to combine.

**Example 6:** Evaluate the following expressions

a) \[-23 + 82 + 23 - 72\]
   - Identify the separate signed terms (integers) in the expression.
   - Combine the opposite integers together.
   - Combine the opposite signed integers together.

b) \[-100 + 798 + 105 - 790\]
   - Identify the separate signed terms (integers) in the expression.
   - Look for and combine opposite signed integers that are “nice”.
   - Combine the integers together.

**You Try It!** Evaluate the following expressions using correct problem structure.

a) \(-5 + 10\)  
   b) \(12 - 8\)  
   c) \(-13 - 8\)

d) \(16 - 12 + 4 - 8\)  
   e) \(-41 - 87 + 83 + 41\)  
   f) \(118 - 819 - 120 + 820\)

You Try It! VIDEO [https://www.youtube.com/watch?v=gDcXHQw-KVM](https://www.youtube.com/watch?v=gDcXHQw-KVM)
Problem Set Directions:
Section problem sets are used to help practice and master topics covered in each section. Each problem should be copied, completed and organized in a separate homework notebook. This homework notebook will be used as a study guide when studying and reviewing for an exam. For each problem in the problem set:
1) Rewrite the directions for each group of problems.
2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

Identify the separate terms in each expression by drawing a vertical dotted line between the terms. How many term(s) are in each expression? DO NOT EVALUATE. Follow Example 1

1. \(-5 + 10(-2) - 4\)  
2. \(-5(10) - 2(5 - 4)\)
3. \(4(x + 3) - (x - 2)\)  
4. \(5 - 10(x - 1)\)
5. \(-x(x + 3)\)  
6. \((-3)(7 - 5) + 11\)

Evaluate or calculate the value of the following expressions. Follow Examples 2-4

7. \(-5 + 10\)  
8. \(12 - 8\)  
9. \(-13 - 8\)  
10. \(-12 - 6 - 10\)
11. \(-1 + 7 + 3 - 12\)  
12. \(-12 + 8 + 10 - 6\)  
13. \(11 + 2 - 10 - 13\)  
14. \(-134 + 354\)
15. \(-11 - 12 + 3 + 22\)  
16. \(-1 - 0 + 3 - 7\)  
17. \(543 - 372\)  
18. \(372 - 543\)

Evaluate or calculate the value of the following expressions. Follow Examples 5-6

19. \(-17 - 32 + 17 + 32\)  
20. \(91 - 10 + 9 - 91\)  
21. \(543 - 132 - 540 + 130\)  
22. \(609 - 543 - 57\)
23. \(5.32 - 4.5 - 5.32\)  
24. \(-\frac{3}{7} + \frac{4}{5} + \frac{3}{7}\)  
25. \(400 + 50 + 8 - 300 - 40 - 7\)
The most basic component of mathematics is the factor. Factors are parts of multiplication, therefore, in the product $3 \times 5$ or $3 \cdot 5$ or $3(5)$ the factors are 3 and 5.

And, since $3 \cdot 5 = 15$, we say 3 and 5 are factors of 15 and $3 \cdot 5$ is the factored form of 15. Note, $(\quad)$ signifies multiplication. We will be using $(\quad)$ to represent factors when working with terms.

In Section 1.3, you will learn how to identify factors within terms to calculate the value of the term. In this section you will learn how to find factors of natural numbers and the greatest common factor between 2 or more natural numbers. You will also learn about factor operators and how they affect the value of factors.

### Objective 1: Finding Prime Factorizations of a Number

One of the most important skills in mathematics is finding all the factors of a composite number. One way to do that is to use the prime factorization of the number.

A prime number is a natural number, other than 1, whose only factors are 1 and itself. The first few prime numbers are:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29,$$ and so on.

A natural number, other than 1, that is not a prime number is called a composite number.

Every composite number can be written as a product of prime numbers. We call this product of prime numbers the prime factorization of the composite number.

In the next few examples, we will find the prime factorization of a natural number using a factor tree.

#### Example 1:
Write each of the following numbers as a product of primes.

a) $40$

First, write 40 as the product of any two natural numbers other than 1.

If the factors are prime, circle them. If not, factor each of these numbers.

Continue this process until all of the factors are prime numbers.

The prime factorization of $40 = 2 \cdot 2 \cdot 2 \cdot 5$
b) 63
   \[ \begin{array}{c}
   9 \cdot 7 \\
   3 \cdot 3 \\
   \end{array} \]

First, write 63 as the product of any two natural numbers other than 1.

If the factors are prime, circle them. If not, factor each of these numbers.

Continue this process until all of the factors are prime numbers.

The prime factorization of 63 is \(3 \cdot 3 \cdot 7\)

**You Try It!** Write each of the following numbers as a product of primes.

a) 36
b) 200

---

**Objective 2** Finding all Factors of a Number and GCF

We need to be able to find and use factors of numbers in everything that we do mathematically. The next example will show you how to use the prime factorization of a number to find all the factors of a number.

**Example 2:** List all the factors of 72.

First, find the prime factorization of 72 → \(2 \cdot 2 \cdot 2 \cdot 3 \cdot 3\)

By finding all combinations of these prime factors we’ll generate all of the factors.

- **72 → 2**: You can begin by grouping one of the 2’s by itself and the rest of the numbers together to get the first pair of factors: 2, 36.
- **72 → 2**: Next, group the first two of the 2’s together to get the next pair of factors: 4, 18.
- **72 → 2**: Group the 2’s and 3’s respectfully to get: 8, 9.
- **72 → 2**: Now, group one of the 3’s and the rest of the numbers to obtain: 24, 3
- **72 → 2**: Lastly, reorder the factors and group one 2 and one 3 together to get 6 and 12.

Once you have found all of the combinations, list the factors along with 1 and the number itself to have the complete list.

Factors of 72 → 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
You Try It! Find all the factors of 60 using its prime factorization.

Notice, in the list of the factors of 72, that the pairs of factors are paired from outside to in in the list.

Factors of 72 → 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

We can use this pattern to generate lists of factors of numbers.

Example 3: List all the factors of 54.

Start the list of factors with the factor pair of 1 and 54….leaving room in the middle for the rest of the potential factor pairs.

Factors of 54 → 1, 54

Check if 2 is a factor: Since, 54 is even, then 2 is a factor of 54 and $54 ÷ 2 = 27$. Insert the factor pair of 2 and 27 into your list.

Factors of 54 → 1, 2, 27, 54

Check 3: $54 ÷ 3 = 18$, insert the factor pair of 3 and 18.

Factors of 54 → 1, 2, 3, 18, 27, 54

Check 4: 4 does not divide 54 evenly.
Check 5: 5 does not divide 54
Check 6: $54 ÷ 6 = 9$, insert the factor pair of 6 and 9.

Factors of 54 → 1, 2, 3, 6, 9, 18, 27, 54

Notice, the factor pairs are coming together in the middle of your list. The only factor pair possible that is left to check is 7 times 8, or $7 \cdot 8 = 56 \neq 54$, therefore the factor list of 54 is complete.

Factors of 54 → 1, 2, 3, 6, 9, 18, 27, 54

Note: Factors divide the number evenly...leaving no remainder. There are some “shortcuts” to checking divisibility of numbers. For instance, if a number ends in an even number, it’s divisible by 2. If the number ends in a 0 or 5, it’s divisible by 5. If the sum of the digits is divisible by 3, the number is divisible by 3. For example, 291 is divisible by 3, since $2 + 9 + 1 = 12$, which is divisible by 3.
You Try It!  Find all the factors of 84 using the listing method from Example 3.

Objective 3  Finding the Greatest Common Factor (GCF)

The GCF is the largest number that divides into both values without a remainder. Or more simply, it’s the largest factor that is common to two or more numbers. In Examples 2 and 3 we found and listed the factors of 72 and 54.

Factors of 72 → 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
Factors of 54 → 1, 2, 3, 6, 9, 18, 27, 54

Therefore, the Greatest Common Factor of 72 and 54 is 18 or

\[ \text{GCF}(72, 54) \rightarrow 18 \]

If we don’t have a list of factors of each number, we can find the GCF by using their prime factorizations.

The GCF of a list of numbers, is a number that contains all the common prime factors of all the numbers in the list. If there are no common factors, then the GCF is 1.

Example 4: Find the Greatest Common Factor of 120 and 45.

\[ 120 \rightarrow 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \]
\[ 45 \rightarrow 2 \cdot 3 \cdot 5 \]

Write the prime factorization of each number.

The GCF contains all of the common prime factors between the numbers.

GCF(120, 45) \rightarrow 3 \cdot 5 = 15

You Try It!  Find the GCF of 24 and 36 using prime factorizations.
Evaluating Factor Operators

Just like there are pronunciation symbols in the dictionary to help us pronounce sounds of letters in words, there are symbols and operators in math that affect the value of factors.

**factor** pronounced \( \text{fāk' tər} \)

We will call these **factor operators** and learn how to evaluate each factor operator after we define them and learn to translate their meaning to English.

**Composition of Mathematics:**  
**Factor Operators** such as the opposite of: \( -() \), absolute value: \(| |\) , square root: \( \sqrt{} \), and exponents (powers), effect the value of individual factors.

**Opposite:**  Two numbers that are the same distance from 0 on the number line but are on opposite sides of 0 are called **opposites** of each other.

<table>
<thead>
<tr>
<th>Translation:</th>
<th>Symbolism:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) “The <strong>opposite</strong> of 2 is -2” <strong>or</strong> ( -(2) \to -2 )</td>
<td></td>
</tr>
<tr>
<td>(b) “The <strong>opposite</strong> of negative 3 is 3” <strong>or</strong> ( -(3) \to 3 )</td>
<td></td>
</tr>
</tbody>
</table>

The **Absolute Value** of a number is defined as its distance from zero on the number line.

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<tbody>
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<td>2</td>
</tr>
<tr>
<td>(b) “The <strong>absolute value</strong> of negative 3 is 3” <strong>or</strong> (</td>
<td>-3</td>
</tr>
</tbody>
</table>

**Exponents or Powers** are used to represent **repeated factors** or **repeated** multiplication.

<table>
<thead>
<tr>
<th>Translation:</th>
<th>Symbolism:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“5 to the second <strong>power</strong> is 25” <strong>or</strong> “5 <strong>squared</strong> is 25” <strong>or</strong> ( 5^2 \to (5)(5) \to 25 )</td>
<td></td>
</tr>
<tr>
<td>“3 to the <strong>fourth power</strong> is 81” <strong>or</strong> “3 to the <strong>exponent</strong> of 4 is 81” <strong>or</strong> ( 3^4 \to (3)(3)(3)(3) \to (9)(9) \to 81 )</td>
<td></td>
</tr>
</tbody>
</table>

*Note: The opposite of zero is \( -(0) \to 0 \) and the absolute value of zero is zero, \( |0| \to 0 \)*

**Problem Structure Tip:**  
Keep track of factors using () as you evaluate the factor value. Practice using \( \to \) and working vertically to keep track of problem solving steps.

*Note: Multiply the repeated factors in an efficient order or grouping. Consider multiplying repeated factors 2 at a time. Try to resist multiplying left to right.*
The **Square Root** (or principal square root) of a number, is the number that is squared to produce that number.

**Symbolism:**

\[
\sqrt{9} \rightarrow 3 \text{ since, } 3^2 \rightarrow 9 \\
\sqrt{100} \rightarrow 10 \text{ since, } 10^2 \rightarrow 100
\]

**Note:** The square root of a negative number is not a real number!

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**Note:** When evaluating square roots, learn to recognize perfect square numbers:

1, 4, 9, 16, 25, 36, 49, ...

How many more perfect square numbers can you list?

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**Discussion:** Why are factors to the second power, “squared” and factors to the third power, “cubed”?

**Example 5:** Each of the following factors involve factor operators. Translate the factor operator to English and then, calculate the value of the factor.

a) \(-(-5)\) Translates to: “The **opposite** of \(-5\)”
   
   \(-5\)

b) \([-3]\) Translates to: “The **absolute value** of \(-3\)”
   
   \(-3\)

Note: The factor value is 3.

c) \(4^3\) Translates to: “4 to the **3rd Power**”
   
   \(64\)

   or “4 **cubed**

   The factor value is 64.

d) \(\sqrt{49}\) Translates to: “The **square root** of 49”

   \(7\)

   The factor value is 7.

---

**You Try It!** Translate the factor operator to English and then, calculate the value of the factor.

a) \(|-5|\)

b) \(2^5\)

c) \(\sqrt{16}\)

d) \(-8\)
Objective 5  Evaluating Grouped Factors

Composition of Mathematics: Factors Operators can also be used as grouping symbols just like parentheses. The expression enclosed by the grouping symbol IS the factor and must be evaluated BEFORE the factor operator can be calculated.

Example 6: Evaluate or calculate the value of the following factors by first evaluating inside the grouping symbols, then evaluating the factor operator.

a) \((-15 + 19)\) Identify the grouping symbol, \((- ( ) )\), “The opposite of…”
   \(- (4)\) Evaluate the expression inside the grouped factor.
   \(- 4\) Evaluate the factor operator

b) \(\sqrt{10 - 8 + 7}\) Identify the grouping symbol, \(\sqrt{ }\), “The square root of…”
   \(\sqrt{17 - 8}\) Evaluate the expression inside the grouped factor.
   \(\sqrt{9}\) Evaluate the factor operator
   \(3\)

c) \((-11 + 17)^2\) Identify the grouping symbol, \((- ( ) )^2\), “A factor squared…”
   \((- 6)^2\) Evaluate the expression inside the grouped factor.
   \(36\) Evaluate the factor operator

You Try It!  Evaluate or calculate the value of the factor for the following grouped factors.

a) \(|3 - 8|\)

b) \((4 - 3)^5\)

c) \(\sqrt{20 - 11}\)

d) \(-(-4 - 10 + 8)\)

e) \((15 - 2 + 1 - 12)^3\)

f) \(\sqrt{-6 + 25 - 10}\)

Problem Structure Tip:
Work vertically, use the factor operator as the grouping symbol as you take steps to evaluate the expression inside. The factor operator symbol goes away only after the operator has been evaluated in the last step.

Tip: You can try these problems BEFORE you watch the video to sort of “test” yourself or you can fill in the steps as you watch the video simulating a lecture.

You Try It! VIDEO
https://www.youtube.com/watch?v=C5MBEgiixbE
Sec 1.2 Problem Set

Problem Set Directions:
Section problem sets are used to help practice and master topics covered in each section. Each problem should be copied, completed and organized in a separate homework notebook. This homework notebook will be used as a study guide when studying and reviewing for an exam. For each problem in the problem set:
1) Rewrite the directions for each group of problems.
2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

Write each natural number as a product of primes.  Follow Example 1
1. 33  2. 60  3. 24  4. 27  5. 20
6. 56  7. 75  8. 84  9. 45  10. 120

List all the factors of the following numbers using the prime factorization.  Follow Example 2
11. 33  12. 60  13. 24  14. 27  15. 20
16. 56  17. 75  18. 84  19. 45  20. 120

List all the factors of the following numbers using the “Listing” method.  Follow Example 3
21. 12  22. 42  23. 28  24. 36  25. 75
26. 76  27. 52  28. 96  29. 50  30. 64

Find the Greatest Common Factor (GCF) of the following numbers.  Follow Example 4
31. 33, 60  32. 36, 45  33. 12, 24, 42  34. 17, 20  35. 45, 120
36. 6,18  37. 8, 16, 24  38. 30, 45, 60  39. 28, 56, 42  40. 300, 450, 600

Translate each factor operator to English, then calculate the value of the factor.  Follow Example 5
41. −(−3)  42. 3⁴  43. (5)³  44. −|−7|  45. √25
46. −(5)  47. 8²  48. |4|  49. √9  50. √100
51. −(−1)  52. (2)⁵  53. |0|  54. √0  55. √7

Evaluate or calculate the value of the following grouped factors.  Follow Example 6
56. −(−9 + 6)  57. (−2 + 9)²  58. |3 − 8|  59. √20 − 11
60. (4 − 3)⁵  61. √8 + 12  62. −(−4 + 9)  63. |−7 + 5|
64. −(−4 − 10 + 8)  65. √6 + 25 − 10  66. (15 − 2 + 1 − 11)⁴  67. |−7 + 5 − 12 + 10|
68. −(72 − 24 + 24)  69. √−9 − 10 + 100  70. (4 + 1)³  71. |131 − 16 − 130|
72. −(−321 − 561)  73. √−735 + 856  74. (−264 + 266)⁶  75. |−542 − 162 − 83|
Section 1.3
Evaluating a Single Term

To evaluate or calculate the value of a term, we must be able to identify and calculate the value of all the factors within the term. In an algebraic term when a number is followed by a variable, multiplication is implied. In the term \(5x\), multiplication is implied between the 5 and the variable \(x\). Therefore, the factors of \(5x\) are 5 and \(x\).

### Objective 1
Identifying Factors within a Term

**Composition of Mathematics:**
A mathematical term consists of one or more factors separated by multiplication and/or grouping symbols consisting of ( ) or factor operators.

#### Example 1:
Identify the factor(s) in each term?

<table>
<thead>
<tr>
<th>Term</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td>(-5)</td>
</tr>
<tr>
<td>(30x)</td>
<td>(30) and (x)</td>
</tr>
<tr>
<td>(-6 \cdot 3(-1))</td>
<td>(-6), (3) and ((-1))</td>
</tr>
<tr>
<td>((-2)^3)</td>
<td>(-2), (-2), (-2) and ([-5])</td>
</tr>
<tr>
<td>(-4(x-1)(x+7))</td>
<td>(-4), ((x-1)) and ((x+7))</td>
</tr>
<tr>
<td>(-2(3-2)^2 \sqrt{x+8})</td>
<td>(-2), ((3-2)), ((3-2)) and (\sqrt{x+8})</td>
</tr>
</tbody>
</table>

From the Composition of Mathematics we know terms are made up of factors. The next example will help you construct or write a term from given factors.

#### Example 2:
Write a term consisting of the given factors.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3, 2, \text{and} -5)</td>
<td>(-3(2)(-5))</td>
</tr>
<tr>
<td>(x, x, x, 12, y, \text{and} y)</td>
<td>(12x^3y^2)</td>
</tr>
<tr>
<td>(\sqrt{11+5}, -2, -3, \text{and} -3)</td>
<td>(-2(-3)^2 \sqrt{11+5})</td>
</tr>
<tr>
<td>(x - 2, x+1, 2, \text{and} x)</td>
<td>(2x(x-2)(x+1))</td>
</tr>
</tbody>
</table>

**Discussion:**
Does it matter what order the factors are in? Why?

**Discussion:**
Why would raised dots NOT work in d)?

\[2 \cdot x \cdot x = 2 \cdot x + 1\]
You Try It! If given a term, list the factors of the term. If given a list of factors, construct or write a term containing those factors.

<table>
<thead>
<tr>
<th>Term</th>
<th>↔</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3</td>
<td>9 (−5\sqrt{10−6}) →</td>
<td></td>
</tr>
<tr>
<td>b) (\sqrt{16}, −3, \text{ and } −3) ←</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) (−2x^3(x−1)^2) →</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) (x−7, x, \text{ and } −7) ←</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Objective 2 Evaluating a Term Containing Integer Factors

By the Composition of Mathematics, a term is made up of factors. Therefore, in order to evaluate or calculate the value of a term we would need to multiply the value of every factor within the term. The value of a factor will be a signed number (integer), therefore we will need to be able to multiply integers together.

Rules for Multiplying Integers

Multiplying same sign integers (both positive or both negative) produces a positive (+) number. Multiplying opposite signed integers (one positive and one negative) produces a negative (−) number.

Multiplying Same Sign Integers

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Multiply 5 and 7</td>
<td>12</td>
</tr>
<tr>
<td>b) Multiply −3 and −2</td>
<td>6</td>
</tr>
</tbody>
</table>

Multiplying Opposite Sign Integers

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>c) Multiply −9 and 12</td>
<td>−108</td>
</tr>
<tr>
<td>d) Multiply 10 and −23</td>
<td>−230</td>
</tr>
</tbody>
</table>

Remember, factors can be multiplied in any order. As long as the term involves the exact same factors, multiplying the factors in any order will produce the same term value.
You can always multiply factors left to right, but to improve your efficiency and accuracy when calculating, identify and group “nice” and convenient factors to multiply together first. This is a very important skill that you will need to use extensively when working with fractional factors, which we cover in section 1.6.

The following example shows ways on how to take advantage of multiplying factors in any order.

**Example 3:** Evaluate or calculate the value of the following terms.

a) \(-2(13)(-5)\) Identify the separate factor values (integers) in the term.

\(10)(13)\) Look for factors of 10, first. Or, group and multiply negative factors first.

130 With a factor of 10, the last multiplication is “nice”.

b) \((-3)^4\) Exponents or powers represent repeated factors of a term.

\((-3)(-3)(-3)(-3)\) Group and multiply the factors 2 at a time keeping track of factor values.

\(9)(9)\) Notice, the value of the term is NOT in ( ).

81

c) \(3\cdot10^3(-7)\) Identify the separate factor values (integers) in the term.

\((-21)(10)(10)\) Notice, the power of 10 is the same as the number of zeros after 1 or \(10^3 \rightarrow 1,000\)

\(10^3 \rightarrow 1,000\) To multiply by 1,000, just attach three 0’s to the end of the number.

\(-21,000\)

The Composition of Mathematics allows you to take advantage of multiplying factors in any order. Try not to just multiply factors from left to right. Practice multiplying factors in any order by identifying and grouping “nice” factors together.

**You Try It!** Evaluate or calculate the value of each term containing integer factors.

a) \(3(-7)\) b) \(-5(-20)\)

c) \(3(-5)(-2)\) d) \(-11(-7)(0)(-3)\)

e) \((-2)^5\)
**Objective 3** Evaluating a Single Term

Again, we are trying to find the value of a term by calculating the product of its factors. So, we must know the value of each factor in the term, BEFORE we can multiply the factors together. Recall from Section 1.2, factor operators affect the value of the factor.

**Composition of Mathematics:**

To evaluate or calculate the value of a term:

1) **Identify** and keep track of the separate factors of the term.
2) **Calculate** the value of each factor (if needed) in **any order**.
   * Factor Operators affect the value of a factor.
3) **Multiply** the factor values together in **any order**.

**Example 4:** Evaluate or calculate the value of the following terms.

a) \(-3\sqrt{16}\)  
   Identify and keep track of the factors of \(-3\) and \(\sqrt{16}\) making up the term.
   Calculate the value of any factor with an operator.
   Multiply the factor values in any order…
   to get the value of the term.

\(-3\)(4) \(-12\)

b) \(3(6-7)^4(-2)^2\)  
   Identify and keep track of the factors of 3, \((6-7)^4\) and \((-2)^2\)
   Calculate the value of any factor with an operator.
   *Grouped factors are evaluated first, BEFORE the factor operator is applied.
   Multiply the factor values in any order…
   to get the value of the term.

\((3)(1)(4)\)

\(12\)

c) \(-(-2)|4-9|\sqrt{25-16}\)  
   Identify and keep track of the factors of \(-(-2)\), \(|4-9|\), and \(\sqrt{25-16}\)
   Calculate the value of the factors in any order.
   Multiply the factor values in any order…
   to get the value of the term.

\((2)(-5)(\sqrt{9})\)

\((2)(5)(3)\)

\((10)(3)\)

\(30\)

Discussion: Why is the term value at the end NOT in parentheses?

Remember, the factors of a term can be written in any order since the values of the factors are eventually multiplied to get the value of the term.

Consider the term in b) can be written and evaluated with the factors in a different order.

\((-2)^2(6-7)^4\)

The factors of the term are in a different order…

\((4)(3)(1)\)

…which does not affect the value of the term once multiplied.
You Try It!  Evaluate the following terms containing multiple factors. Practice using correct problem structure, working vertically and using parentheses to keep track of individual factors.

a) \((-2)^2 \mid -2 \mid\) 

b) \(|6-9|\sqrt{16+9}\)

c) \((-1)^2(-5+8)^2\sqrt{36}\) 

d) \((15-10)^2(-3-7)^2\)

e) \(-5\sqrt{9}(-2)^2 \mid -5 \mid\) 

f) \(-3 \mid -2-5 \mid (-5+4)^3\)
Sec 1.3 Problem Set

Problem Set Directions:
Section problem sets are used to help practice and master topics covered in each section. Each problem should be copied, completed and organized in a separate homework notebook. This homework notebook will be used as a study guide when studying and reviewing for an exam. For each problem in the problem set:
1) Rewrite the directions for each group of problems.
2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

List the individual factors within each term. DO NOT EVALUATE

Follow Example 1
1. \((-2)^3\) 2. \(-5\sqrt{5}\) 3. \(5x^2y\) 4. \(3\alpha(a+2)\) 5. \(-5\sqrt{25-16}\)
6. \(-3x^2(x+1)\) 7. \(-2(3x+5)(x-1)\) 8. \((-12 + 5)|6-9|\) 9. \(-5\sqrt{5}(5-8)^2\) 10. \(3x(x-2)(x^2+2x+1)\)

Write a term containing the given factors. DO NOT EVALUATE

Follow Example 2
11. \(-5\) and \(-3\) 12. \(
\sqrt{64}\), and 8 13. \(10, x, x, \text{and } x\) 14. \([-3-5], -1, \text{and } \sqrt{5}\) 15. \(-3, \text{and } x+2\)
16. \(-2, -2, -2, x-2, x-2, \text{and } x\) 17. \(x+1, -3, \text{and } \sqrt{x-4}\) 18. \(2x^2+x-5 \text{ and } x-5\)

Evaluate or calculate the value of the following terms using correct problem structure. Follow Examples 3 and 4
19. \(-5(10)\) 20. \(-3(-2)\) 21. \((-12)\) 22. \(-(-1)(5)\) 23. \((-5+4)(10-11)(-2+7)\)
24. \(2(-5)\) 25. \(-3(-4)(-2)\) 26. \(-2(4)(-2)(5)\) 27. \(-12(11)(0)(-5)\) 28. \((-7-2)(-12+12)\)
29. \((-2)^3\) 30. \(-5(-2)^2(3)^2\) 31. \(-7|-3|\) 32. \((-12+5)|6-9|\) 33. \(-5\sqrt{5}\)
34. \(-5\sqrt{25-16}\) 35. \(-2(-5+10)\sqrt{25-16}\) 36. \(-2(-3)^2\) 37. \(-5\sqrt{5}(5-8)^2\) 38. \((-2)^4\sqrt{9+27}\)
39. \(\sqrt{4}\sqrt{(-3+3)^2}\) 40. \(-3(-5+7)^2\) 41. \((-3+5)^3(3-5)^3\) 42. \((-1)^2(5-6)^3|7-8|\)
Section 1.4  
Evaluating Expressions and Problem Structure

Now that we know how to evaluate or calculate the value of a single term in mathematics, we are ready to evaluate or calculate the value of an expression that contains multiple terms. Since all evaluations or calculations that we do in mathematics is based on the composition of mathematics, then we need to be able to identify all the components of an expression. This is called “reading” the composition.

Objective 1  
Reading the Composition of an Expression

Before we can evaluate any expression, we must be able to “READ” the composition of an expression by identifying the separate terms in the expression and the separate factors within each term. We also say that, the separate terms and factors in an expression, make up the components of the expression.

Example 1:  
What is the composition of the following expressions?

a) \(3 - 5(2) + 4^2\)  
\(= 3 - 10 + 16\)

First, identify the separate terms using a vertical dotted line.  
Then, identify the separate factors within each term.

b) \(-2\sqrt{16} + 5|9 - 3| - 7 + (-3)^2\)  
\(= -8 + 5(6) - 7 + 9\)

This expression has four terms.  
Recall, exponents represent repeated factors.
Discussion: Could the grouped factors \((x+2)\) and \((x^2+2)\) be considered common factors?

**Problem Structure Tip:** Identify the composition of the expression (term(s) and factor(s)) as you rewrite the problem from the textbook to your homework notebook.

**Objective 2**  
**Problem Structure**

By the composition of mathematics, if we are trying to find the value of an expression, we must know the value of each term in the expression, BEFORE we can combine the terms together.

**Composition of Mathematics:**  
To evaluate or calculate the value of an expression:
1. Identify and keep track of the separate terms of the expression.
2. Calculate the value of each term (if needed) in any order.
3. Combine the term values together in any order.

We need to use good problem structure to keep track of the separate term values as we evaluate the expression and to keep track of the separate factor values as we evaluate each term.

**Example 2:** Evaluate or calculate the value of the following expressions.

a)  
\[3 - 5(2) + 4^2\]  
Identify and keep track of the separate terms in the expression.  
Calculate the value of each term (if needed) in any order.  
Combine the term values in any order…  
...to get the value of the expression.

b)  
\[\sqrt{16} - (-4) - 7 + 3^2\]  
Identify and keep track of the separate terms in the expression.  
Calculate the value of each term (if needed) in any order.  
Combine the term values in any order…  
...to get the value of the expression.

Notice, the term in b) \((-4)\) translates to “The opposite of negative 4” which calculates to positive 4. This would be exactly the same as if we multiplied \(-4\) by \(-1\).

Therefore, the opposite of a number is the same as \(-1\) times the number.

\[-(-4) \rightarrow (-1)(-4)\]

**Composition of Mathematics:**  
The opposite of a factor is the same as a factor of \((-1)\) in the term.
Identifying Factors of $-1$

Since terms in an expression are separated by $+$ or $-$ outside of grouping symbols, then we need to be able to understand how to evaluate the opposites of factors involving factor operators.

**Example 3:** Evaluate or calculate the value of each term.

On the left, we will use the definition of the opposite of a factor with an operator to calculate the value of the term. On the right, the same expression can be evaluated as a term containing a factor of $(-1)$.

\[ \text{a) } -\sqrt{16} \]

“The opposite of the square root of $16$”

- Evaluate the factor operator.
  - Then, take the opposite of the factor.

\[ = -4 \]

Identify the factor of $(-1)$

\[ \text{or } \sqrt{16} \]

- Calculate each factor value.
  - Multiply the factors.

\[ = -4 \]

\[ \text{b) } -|{-2}| \]

“The opposite of the absolute value of $-2$”

- Evaluate the factor operator.
  - Then, take the opposite of the factor.

\[ = 2 \]

Identify the factor of $(-1)$

\[ \text{or } |{-2}| \]

- Calculate each factor value.
  - Multiply the factors.

\[ = 2 \]

\[ \text{c) } -(5)^2 \]

“The opposite of negative $5$ squared”

- Evaluate the factor operator.
  - Then, take the opposite of the factor.

\[ = 25 \]

Identify the factor of $(-1)$

\[ \text{or } (5)^2 \]

- Calculate each factor value.
  - Multiply the factors.

\[ = 25 \]

\[ \text{d) } -5^2 \]

“The opposite of $5$ squared”

- Evaluate the factor operator.
  - Then, take the opposite of the factor.

\[ = 25 \]

Identify the factor of $(-1)$

\[ \text{or } (5)^2 \]

- Calculate each factor value.
  - Multiply the factors.

\[ = 25 \]

With respect to the Composition of Mathematics, we want to identify and evaluate individual factors within a term. So, we want to practice recognizing the “opposite of” as a factor of $-1$ in the term.

We need to be very careful when interpreting the factor operators of exponents. The other factor operators (absolute value and square roots) act as grouping symbols, but exponents or powers need ( ) in order to define the factor or base of its exponent.

“Negative $2$ to the $4$th power”

\[ (-2)^4 \]

“Watch Out!”

“The opposite of $2$ to the $4$th power.”

\[ 2^4 \]

Within expressions, the composition of each of these are defining different components.

\[ -5(-2)^4 \]

This expression is one term... with $(-2)^4$ representing a factor of the term.

\[ -5 \]

This expression has two terms... with $-2^4$ representing one of the terms of the expression.

\[ (-5)(16) \]

\[ -80 \]

\[ -21 \]
The next examples will show how we can **keep track** of factors within terms and terms within expressions as we evaluate or calculate the value of each expression.

**Example 4:** Evaluate or calculate the value of the following expressions.

a) \( \frac{8}{11} - \frac{3}{10} \)

This expression has two terms to keep track of as you calculate its value.

\[
\begin{align*}
(1) & \quad \frac{8}{11} \\
(2) & \quad \frac{3}{10} \\
\text{Combine the terms in any order.}
\end{align*}
\]

b) \( 90 - (6 - 4)^2 \)

This expression has two terms to keep track of as you calculate its value.

\[
\begin{align*}
(1) & \quad -10 \\
(2) & \quad 100 \\
\text{Combine the term values in any order.}
\end{align*}
\]

**Problem Structure Tip:** Write mathematics using a vertical format.

Just like the **vertical format** is used while adding and multiplying numbers with multiple digits, we use the vertical format while evaluating or calculating the value of expressions. The vertical format enables us to keep track of the separate terms in the expression and the separate factors within each term as we evaluate. Also, we can follow vertical columns easier to help track mistakes if we need to find them.

**Example 5:** Evaluate each term below using the correct problem structure.

a) \( (-2)^3 \sqrt{90 - 9} \)

Identify and keep track of the factors as you calculate their values.

\[
\begin{align*}
(1) & \quad 4 \quad \sqrt{81} \\
(2) & \quad 9 \\
\text{Multiply the factor values in any order to get the value of the term.}
\end{align*}
\]

b) \( -(8 - 11) \times 3(-2) \)

Identify and keep track of the factors as you calculate their values.

\[
\begin{align*}
(1) & \quad -10 \\
(2) & \quad -6 \\
\text{Multiply the factor values in any order to get the value of the term.}
\end{align*}
\]

Both terms in example 5 took multiple steps to evaluate. What if we needed to evaluate an expression that contains these two terms?

**Example 6:** Evaluate the following expression using the correct problem structure.

\[
\begin{align*}
(-2)^3 \sqrt{90 - 9} \times (8 - 11) \times 3(-2) \\
(1) & \quad 4 \quad \sqrt{81} \\
(2) & \quad (1)(-10) \\
(3) & \quad 3(6) \\
\text{Combine the term values in any order…}
\end{align*}
\]

Both terms in example 5 took multiple steps to evaluate. What if we needed to evaluate an expression that contains these two terms?
**You Try It!** Evaluate or simplify the following expressions using correct problem structure.

**Practice using correct problem structure**, working vertically and using **dotted lines** to keep track of separate **terms** and ( ) to keep track of separate **factors** as you simplify the expression.

a) \[7 - 4(-2)\]

b) \[(-4)^2 - 3(-4) + 2\]

c) \[-5^2 + 2(-6 + 3)^2\]

d) \[2\sqrt{25} - \sqrt{-4 + 8} + 4\sqrt{4}\]

e) \[3(-2)^2 + 5\sqrt{16 - 7} - 3| -2|\]

f) \[-4^2 + 5|3 - 7| + \sqrt{64} + \sqrt{16}\]

---

**Objective 3**

**Order of Operations**

You are now able to READ mathematics by identifying separate terms within expressions and separate factors within each term. Now you can use the composition of mathematics to remember the order in which you calculate the value of each component.

Therefore, the **Composition of Mathematics** defines order of operations.

Ever wonder why PEMDAS (Parentheses, Exponents, Multiply and Divide, Add and Subtract) defines order of operations? Because it follows the **Composition of Mathematics** (Grouped Factors and Factor Operators, Multiply Factors, Combine Terms)

---

The value of an expression is the sum of all its term values. The value of a term is the product of all its factor values. Factor operators affect the factor value.
If you know the value of every factor, you can find the value of the term containing those factors. And, if you know the value of every term, you can find the value of an expression containing those terms.

**Composition of Mathematics:** Order of Operations
1. Evaluate all grouped factors and factor operators.
2. Multiply factor values in any order.
3. Combine term values in any order.

**Order of Operations**

3. Calculate the expression value by combining all its term values.

2. Calculate all term values in any order as the product of its factor values.

1. Calculate all factor values in each term of the expression in any order.

The following example shows how the Composition of Mathematics defines order of operations which guides the problem structure and steps to evaluating.

**Example 7:** Evaluate the following expressions using the Composition of Mathematics.

a) \(-(-3)^3 \sqrt{25-21}\)

Read the Composition: This expression is one term.

\((-1)(-27) \sqrt{4}\)

Identify and keep track of the factors as you calculate their values.

Multiply the factor values in any order to get the value of the term.

\((-1)(-27) (2)\)

\(27 (2)\)

\(54\)

**Hint:** Notice how the expressions in a) and b) might look very similar at first glance. But, the composition of the expressions are very different.

b) \(-(-3)^3 - \sqrt{25-21}\)

Read the Composition: This expression has two terms to keep track of as you calculate their values.

Identify and keep track of the factors in each term as you calculate their value.

Multiply the factor values in any order to get the value of each term.

Combine the term values in any order to get the value of the expression.

\((-1)(-27) \sqrt{4}\)

\((-1)(2)\)

\(27 - 2\)

\(25\)

c) \(-5^2 - 10\sqrt{7} + 32 + 2(5 - 8)^2\)

Read the Composition: This expression has three terms to keep track of as you calculate their values.

Identify and keep track of the factors in each term as you calculate their value.

Multiply the factor values in any order...

... to get the value of each term.

Combine the term values in any order...

... to get the value of the expression.

\((-1)(5^2) (10) \sqrt{25}\)

\((-1)(25) (10)(5) (2)(9)\)

\(-25 - 50 + 18\)

\(-75 + 18\)

\(-57\)
The *Composition of Mathematics* provides a **common structure** that we use to…

- distinguish components of and explain differences between expressions in mathematics. This will allow us to explain, in terms of composition, what type of mistake we made (numeric calculation or a compositional mistake).
- define **order of operations** which guides problem structure when evaluating or calculating values of factors, terms and expressions.

---

**Composition of Mathematics:**
Identifying the composition of an expression is the single most important understanding of all of mathematics.

**“READ” the composition of the expression BEFORE calculating any part of it!**

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**You Try It!** Evaluate the following expressions using the *Composition of Mathematics*

a) \[ -(-2)^2 (5 - 6)^2 (-3)^2 \]

b) \[ -(-2)^2 + (5 - 6)^2 - 3^2 \]

c) \[ |-2| - 5^2 - 2\sqrt{25} \]

d) \[ |-2| - \left(5^2 - 2\sqrt{25}\right) \]
Sec 1.4 Problem Set

Problem Set Directions:
Section problem sets are used to help practice and master topics covered in each section. Each problem should be copied, completed and organized in a separate homework notebook. This homework notebook will be used as a study guide when studying and reviewing for an exam. For each problem in the problem set:
1) Rewrite the directions for each group of problems.
2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

Define the composition of each expression below. Use a vertical dotted line to separate the terms and write the factors of each term in parentheses ( ) if not already grouped. DO NOT SIMPLIFY  
Follow Example 1
1. \(3(-2) - (-7)\)  
2. \(-2^3 - 4\)  
3. \(2 - (-5)^2 + 3\sqrt{4}\)  
4. \(-(3)^3 \sqrt{25 - 21}\)  
5. \(-(3)^3 + \sqrt{25 - 21}\)  
6. \(3(-5 + 4) - 2(10 - 11)\)

Evaluate the following expressions using correct problem structure. Follow Examples 2-6
7. \(-8 + 4(3)\)  
8. \(3(-2) - (-7)\)  
9. \(7 - 2(5 - 3)\)  
10. \(5 + 4(-2) - 2(5) - 9\)  
11. \(5 + 4(-3)\)  
12. \(-(2)(5) + (-7)(-1)\)  
13. \(-2(5 - 3) + 7\)  
14. \(-4 - 2\cdot3\)  
15. \(-(2)^3 - 4\)  
16. \(-4 + (-2)^3\)  
17. \(-2^3 - 4\)  
18. \(-4 - 2^3\)  
19. \(-2 - 3\sqrt{4}\)  
20. \(3(0)^2 - (0) + 5\)  
21. \(17 - 2(-3)^2\)  
22. \(3 + 2(-1) + (-1)^2\)  
23. \(3(-5 + 4) - 2(10 - 11)\)  
24. \(-4 + (-5)^2\)  
25. \(-4 - 5^2\)  
26. \(-3(-2)^3 - (-2) + (-2)\)  
27. \(-3[3 + 7] + 2[7 - 2]\)  
28. \(2 - (-5)^2 + 3\sqrt{4}\)  
29. \(3^2 - (1 + 5)\)  
30. \(3\sqrt{19} - 3\sqrt{49} + 2\sqrt{9}\)  
31. \(4[2] - 3\sqrt{4} + 5^2\)  
32. \(-4^2 - 3^2 - 2^2\)  
33. \(-(4)^2 + (-3)^2 + (-2)^2\)  
34. \(\sqrt{-11 + 15} - \sqrt{29 - 4}\)

Evaluate the following expressions using the Composition of Mathematics. Follow Example 7
35. a) \(-5 - (-4)\)  
   b) \(-5(-4)\)  
36. a) \(-2\sqrt{-3} + 12\)  
   b) \(-2 + \sqrt{-3} + 12\)  
37. a) \(-3^3(-3)^2\)  
   b) \((-3)^2 - 3^2\)  
38. a) \(-(3)^3 \sqrt{25 - 21}\)  
   b) \(-(3)^3 + \sqrt{25 - 21}\)  
39. a) \(-3 - 2(5 - 3)\)  
   b) \(-3 - 2)(5 - 3)\)  
40. a) \(2\sqrt{25 - \sqrt{5}}\)  
   b) \(2\sqrt{25 - 9}\)  
41. a) \(-[7 - 9] + \sqrt{25 - 4^2}\)  
   b) \(-[7 - 9] + \sqrt{25 - 4^2}\)  
42. a) \(-2 - 4(-2)^3 - 10\sqrt{5}\)  
   b) \(-2 - 4((-2)^3 - 10)\sqrt{5}\)  
43. a) \(7 - 5\sqrt{16 - 5 - 7}\)  
   b) \((7 - 5)\sqrt{16} - 5 - 7\)  
   c) \(7 - 5\sqrt{16 - 5 - 7}\)
Algebra is a language used to define numeric values and explain quantitative process. Or we can say, algebra is a language that explains what we need to do with certain numbers to calculate desired values. Algebraic expressions are examples of how algebra does this. In this section, you will learn how to evaluate algebraic expressions for values of the variable(s), use the Distributive Property to change the composition of an algebraic expression from factored form to expanded form and vice versa and translate the quantitative process from English to algebra and vice versa.

### Evaluating Algebraic Expressions

Below is some terminology associated with algebraic expressions.

Consider the algebraic expression: \( 7x^2 - x + 6 \)

The variable terms are \( 7x^2 \) and \(-x\) and the constant term is \( +6 \). The coefficient of a term is/are the numeric factor(s) of the term. Therefore, the coefficient of the variable term \( 7x^2 \) is \( 7 \) and the coefficient of the variable term \(-x\) is \(-1\) since \(-x \rightarrow (-1)(x)\).

The variable(s) in an algebraic expression represent numbers or are placeholders for numbers. If we replace or substitute the variable(s) in the expression with a value, we say we are evaluating the algebraic expression for the given variable.

#### Example 1: Evaluate the following expression for \( x = 3 \), \( y = -5 \) and \( z = 25 \).

**a)** \(-2x^2 - y + \sqrt{z}\)

| \((-2)(3)^2\) | \((-1)(-5)\) | \(+\sqrt{25}\) |
| (-2) (9) | +5 | +5 |
| -18 | | |
| -18 | +10 | |
| -8 | |

**Composition:** Identify the separate terms and factors in each term. Replace the variable factors in each term with the given values. Calculate the value of each term. Combine the term values in any order.

**Hint:** \(-x\) can also be interpreted as “The opposite of a number”, which is the same as a factor of \((-1)\) in the term.

**b)** \(-(1-x)^2 - y^2 + z\sqrt{z}\)

| \((-1)(1-(3))\) | \((-1)(-5)\) | \(+\sqrt{25}\sqrt{25}\) |
| (-1) (25) | +25 (25) | (5) |
| -4 | -25 | +125 |
| -4 | | +100 |

**Composition:** Identify the separate terms and factors in each term. Replace the variable factors in each term with the given values. Calculate the value of each term. Combine the term values in any order.
You Try It!  Evaluate the following expressions for $x = -1$, $y = 3$ and $z = -2$

a) $x - 4y - z$

b) $2x^2 - 3x + 5$

c) $7 - \sqrt{y - 6x}$

d) $-(y - x)^2 - z^2$

**Objective 2**  Combining Like Terms

The variable factor(s) of an algebraic term define or name the term.

**Composition of Mathematics:**

*Like terms* contain the exact same variable factors raised to the same powers.

Below are some examples of terms that are alike and terms that are not.

<table>
<thead>
<tr>
<th>Like Terms</th>
<th>Unlike Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x$ and $5x$</td>
<td>Same variables</td>
</tr>
<tr>
<td>$-10p^2$ and $25p^2$</td>
<td>Same variable to the same power</td>
</tr>
<tr>
<td>$-cd^2$ and $3cd^2$</td>
<td>Same variables to the same powers.</td>
</tr>
<tr>
<td>$2x$ and $5y$</td>
<td>Different variables</td>
</tr>
<tr>
<td>$-10p$ and $25p^2$</td>
<td>Same variable, but different powers.</td>
</tr>
<tr>
<td>$-c^2d$ and $3c^2$</td>
<td>The variables are not the same</td>
</tr>
</tbody>
</table>

By the fundamental property of addition, we are able to combine like terms. Therefore, we should be able to combine $2x$ and $5x$.

Since multiplication is repeated addition, then each term can be expanded into a group of like things. Therefore, the $2x$ term represents a group of $2$, $x$’s and the $5x$ term represents a group of $5$, $x$’s.

The coefficients act as “term counters” and represent the total number of variable terms in each grouping. Therefore, the total grouping of $7$, $x$’s, can be written as $7x$.  

*Hint:* Think of any multiplication as representing a group of like things. Therefore, a group of 3 cars can be represented as $3c$. Or, a group of five teachers can be represented as $5T$.  

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Example 2: Combine the like terms in each expression.

a) \(-2x + 7x - 5x\) Recognize like terms
b) \(5 - y + 6y + 4\) Recognize like terms

Combine coefficients
Combine coefficients of variable terms
and keep track of combined terms.

c) \(3a^2 - 7a^2\) Recognize like terms
d) \(3b^2 - 6b + 5b^2 - 7\) Recognize like terms

Combine coefficients
Combine coefficients of variable terms
and keep track of combined terms.

Objective 3 Expanded Form – The Distributive Property

A grouped factor is a factor with or without an operator that contains more than one term. In this section, we will only consider grouped factors without an operator.

Recognizing grouped factors in a term.

<table>
<thead>
<tr>
<th>Term</th>
<th>Multiple Term Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3(x - 2))</td>
<td>factor ((x - 2)) contains 2 terms.</td>
</tr>
<tr>
<td>(-4(3a - 7))</td>
<td>factor ((3a - 7)) contains 2 terms.</td>
</tr>
<tr>
<td>(12(5 - 2y + z))</td>
<td>factor ((5 - 2y + z)) contains 3 terms.</td>
</tr>
<tr>
<td>(- (x^2 - 3x + 5))</td>
<td>factor ((x^2 - 3x + 5)) contains 3 terms.</td>
</tr>
</tbody>
</table>

The composition of an algebraic expression is classified into two forms.

Factored Form – An expression composed of a single term.

Expanded Form – An expression composed of two or more terms or an expression composed of a single term containing NO grouped factors

Example 3: Classify the composition of each expression as factored form, expanded form or both.

a) \(-3(x - 2)\) Composition: single term containing a grouped factor
Factored Form

b) \(-3x + 6\) Composition: two terms
Expanded Form

c) \(7(x + 1) - (x + 2)\) Composition: two terms
Expanded Form

d) \(7(x + 1)(x + 2)\) Composition: single term containing grouped factors
Factored Form

e) \(-3x^2 y\) Composition: single term containing NO grouped factors
Factored or Expanded Form

f) \((2 - 3x) - (x^2 - 2x + 1) - 5\) Composition: 3 terms
Expanded Form
Recall by the *Fundamental Principle of Multiplication*, that multiplication is *performed term-by-term* in any order. This was seen by writing numbers in expanded form.

### Multiply Digit-by-Digit or Multiply Term-by-Term

<table>
<thead>
<tr>
<th>$13 \times 3 \rightarrow 23$</th>
<th>$3(10 + 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\times 3$</td>
<td>$30 + 9$</td>
</tr>
<tr>
<td>$39$</td>
<td>$39$</td>
</tr>
</tbody>
</table>

The **Distributive Property** is the same as the *Fundamental Principle of Multiplication*.

### The Distributive Property:

Grouped factors with NO operators must be multiplied *term-by-term*.

\[ a(b + c) \rightarrow ab + ac \]

### Example 4:

Multiply by applying the **Distributive Property** to expand the following terms containing grouped factors.

#### a) \(-5(2x - 3)\)

**Composition:** Identify a **single term** with the **grouped factor** *(factored form)*

\[ (-5)(2x) \rightarrow (-5)(-3) \]

**Distribute** the factor of \((-5)\) to both **terms** in the **grouped factor**.

Multiply factors in each expanded term. *(expanded form)*

\[-10x + 15\]

**Note:** When multiplying, we say we are **distributing a common factor** to every term.

#### b) \((-x^2 - 5x + 7)\)

**Composition:** Identify a **single term** with the **grouped factor** *(factored form)*

\[ (-1)(x^2) \rightarrow (-1)(-5x) \rightarrow (-1)(7) \]

**Distribute** the factor of \((-1)\) to all **terms** in the **grouped factor**.

Multiply factors in each expanded term. *(expanded form)*

\[-x^2 + 5x - 7\]

We really only need to keep track of the expanded terms, we do not need to actually show each distribution. Arrows can be used to remind us of the distributions that were made while multiplying.

#### c) \(-3(2x - y + 10)\)

**Composition:** Identify a **single term** with the **grouped factor**.

**Distribute** the factor of \((-3)\) to all **terms** in the **grouped factor**...

…multiplying factors in each expanded term.

### Problem Structure Tip:

*Work vertically lining up the expanded terms under the grouped factor to keep track of the expanded terms.*

### Composition of Mathematics:

When multiplying, the **Distributive Property** allows us to **change the composition** of an expression from **factored form** to **expanded form**.
**You Try It!** Expand (Distribute) the following terms containing a grouped factor.

a) \(-3(x - 5)\)  
b) \(-3(5x^2 - x - 7)\)

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**Objective 4** Simplifying Algebraic Expressions

**Composition of Mathematics:**
To simplify an algebraic expression we need to expand any terms containing grouped factors (if needed) and combine like terms.

**Example 5:** Simplify the following expressions.

a) \(7 - 2(3x - 5)\)  
Identify the separate terms in the expression.
Expand terms containing grouped factors by distributing.
Combine like terms in any order.

\[
7 - 2(3x - 5) \\
\quad = 7 - 6x + 10 \\
\quad = -6x + 17
\]

b) \(2(5a + 4) - 3(a - 1) - (8 - 6a)\)  
Identify the separate terms in the expression.
Expand terms containing grouped factors by distributing. *Notice the factor of \((-1)\) in the third term.
Combine like terms in any order.

\[
2(5a + 4) - 3(a - 1) - (8 - 6a) \\
\quad = 10a + 8 - 3a + 3 - 8 + 6a \\
\quad = 13a + 3
\]

**You Try It!** Simplify the following expressions.

a) \(7 - (3x - 5) + 3(4 - x)\)  
b) \(3(2x^2 - x + 4) - (5x^2 - 3x + 7)\)
Translating Algebraic Expressions

Algebra is a language that is used to define mathematical relationships and procedures. In order to solve application problems using algebra, we need to be able to translate word phrases into mathematical expressions.

Here are some common English words and phrases with their mathematical translations.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Expression</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>( a + b )</td>
<td>the sum of ( a ) and ( b ) ( a ) increased by ( b ) ( b ) more than ( a ) ( a ) added to ( b ) ( a ) plus ( b )</td>
</tr>
<tr>
<td>Subtraction</td>
<td>( a - b )</td>
<td>the difference of ( a ) and ( b ) ( b ) subtracted from ( a ) ( a ) decreased by ( b ) ( b ) less than ( a )</td>
</tr>
<tr>
<td>Multiplication</td>
<td>( ab )</td>
<td>the product of ( a ) and ( b ) ( a ) multiplied by ( b ) ( a ) of ( b )</td>
</tr>
<tr>
<td>Division</td>
<td>( \frac{a}{b} )</td>
<td>the quotient of ( a ) and ( b ) ( a ) divided by ( b ) ( \text{the ratio of } a \text{ to } b )</td>
</tr>
</tbody>
</table>

**Example 6:** Translate each phrase into an algebraic expression. Use the variable \( x \) to represent the unknown number.

- **Word Phrase** → **Algebraic Expression**
  - a. A number decreased by 2 → \( x - 2 \)
  - b. Three more than a number → \( x + 3 \)
  - c. The product of 6 and a number → \( 6x \)
  - d. The difference of a number and 4 → \( x - 4 \)
  - e. The quotient of a number and 2 → \( \frac{x}{2} \)
  - f. The sum of twice a number and 5 → \( 2x + 5 \)
  - g. The difference of 3 times a number and 4 → \( 3x - 4 \)
  - h. One more than 4 times a number → \( 4x + 1 \)
  - i. Twice the sum of a number and 5 → \( 2(x + 5) \)
  - j. Three times the difference of a number and 4 → \( 3(x - 4) \)

**Example 7:** Translate each algebraic expression into an English phrase. Let \( x \) represent a number and use the words *sum* and *difference* when appropriate.

- **Algebraic Expression** → **Word Phrase**
  - a. \( x - (-2) \) → The difference of a number and negative 2
  - b. \( 20x + 35 \) → The sum of twenty times a number and 35
  - c. \( 6(9 - x) \) → Six times the difference of a 9 and a number
  - d. \( \frac{x + 4}{5} \) → The sum of a number and 4, divided by 5
You Try It!  Translate each phrase into an algebraic expression. Use the variable $x$ to represent the unknown number.

a) Eight less than a number  →

b) The difference of twice a number and 10  →

c) Six less than the product of a number and 7  →

Translate each algebraic expression into an English phrase. Let $x$ represent a number and use the words sum and difference when appropriate.

d) $-8 + x$  →

e) $-4x + 7$  →

f) $\frac{1}{2}(x - 12)$  →
Sec 1.5 Problem Set

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2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

Evaluate the following expressions for \( x = -2, \ y = 4 \) and \( z = -1 \) Follow Example 1
1. \( x + y + z \) 2. \( x - y - z \) 3. \( 2x - 3y - 4z \) 4. \( 5x - y + 4z \)
5. \( x^2 - y \) 6. \( x^2 + z \) 7. \( 2x^2 - 3x + 5 \) 8. \( 3z^2 - 4z + 2 \)
9. \( 3xz + yz \) 10. \( 3x^2 + 2x^2z \) 11. \( x^2 - y^2 \) 12. \((x + y)^2 \)
13. \( x^2 + xy + y^2 \) 14. \(-2|x| - 3|y| - 5|z| \) 15. \(2 - 3|x| + 5\sqrt{y} \) 16. \(-x^3 + 3|z| - \sqrt{y} \)

Simplify each expression by combining like terms. Follow Example 2
17. \(3x + 5x \) 18. \(5y - 3y \) 19. \(4a + 5a - 3a \) 20. \(6 - 2x - 8 + 5x \) 21. \(-x + 4y + 7x - 9y \)
22. \(2n - 5n + 7 \) 23. \(-2x^2 - 3x + 5x^2 + 3 \) 24. \(7x^2 - 5x + 10 - 6x^2 - 2x - 12 \)

Classify the composition of each expression as factored form, expanded form or both. Follow Example 3
25. \(-3(4x - 1) \) 26. \(-3 + 4(x - 1) \) 27. \(3x(2x - 3)(4x - 1) \) 28. \(-9a^2b \) 29. \(a^4 - 9a^2b - 3b^2 \)

Expand the following terms using the Distributive Property. How many terms are in your expression after expanding? Follow Example 4
30. \(3(x + 4) \) 31. \(5(2x + 1) \) 32. \(-3(4x - 1) \) 33. \(-(2x - 3) \) 34. \(-5x + 3) \)
35. \(2(z^2 - 3z + 5) \) 36. \(-3(5x^2 - x - 7) \) 37. \(-(4y^2 + 3y - 5) \)

Simplify each expression. Follow Example 5
38. \(2(x + 4) + 3 \) 39. \(3 + 2(x + 4) \) 40. \(5(6 - y) - 2 \) 41. \(3y - 2(y - 5) \) 42. \(2(x - 3) + 4(x + 2) \)
43. \(5x - (x - 9) \) 44. \(3 + 2(x - 7) + 5x \) 45. \(5 - 3(2x - 7) + 4x \) 46. \(6(2x - 1) - 12x \)
47. \(-7(y - 2) + 6 \) 48. \(5y - 2(y - 1) + 3 \) 49. \(-6(-2x - 1) - (-x - 3) \) 50. \(7 - 5(2x - 1) + 10x \)
51. \(3(n^2 - 5n + 7) - 2(n^2 + n - 3) \) 52. \(-2(x^2 - 3) + 5(x^2 + 3) \) 53. \(7 - 3(2m - 5m) - 15m \)
54. \(5 - 3(2x - 7) + 4x - 6(-2x - 1) - (-x - 3) \) 55. \((-4 - n^2) + 3(n^2 - 5n + 7) - 2(n^2 + n - 3) - 6(n - 2) \)

Translate each phrase into an algebraic expression. Use the variable \( x \) to represent the unknown number. Follow Example 6
56. the sum of ten and 6 times a number 57. a number decreased by 12
58. the product of negative 7 and a number 59. the sum of a number and seven, divided by 4
60. eight less than a number 61. ten times the difference of 3 and a number
62. fifteen more than twice a number 63. double the sum of a number and 11
64. five times the sum a number and 2 65. 5 more than the quotient of a number and 3

Translate each algebraic expression into an English phrase. Let \( x \) represent a number and use the words sum and difference when appropriate. Follow Example 7
66. \(x + 7 \) 67. \(-5x \) 68. \(2x + 1 \) 69. \(3x - 1 \) 70. \(12 - 3x \)
71. \(-2(x + 5) \) 72. \(2(1 - x) \) 73. \(3(2x + 9) \) 74. \(\frac{3}{x} \) 75. \(\frac{x - 3}{2x} \)

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Section 1.6
Fractional Factors

In this section, we will only look at single terms containing fractions as factors. We will learn to identify the separate factors contained in these terms and how to take advantage of the fact that we can multiply factors in any order to conveniently multiply the “fractional factors” together within a fractional term.

Objective 1 Multiplying Fractional Factors

Multiplying Fractional Factors

To multiply two fractions, multiply the numerators and multiply the denominators. If \(a, b, c, \) and \(d\) represent factors, and \(b\) and \(d\) are not 0, we have

\[
\frac{a \cdot c}{b \cdot d} = \frac{a \cdot c}{b \cdot d} \quad \text{or} \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{(a)(c)}{(b)(d)}
\]

Example 1: Multiply the following fractional factors together.

a) \(\frac{2}{3} \cdot \frac{2}{7} \rightarrow \frac{2}{3} \cdot \frac{2}{7} \rightarrow \frac{4}{21}\) Multiply the numerators and multiply the denominators.

b) \(-6 \cdot \frac{-2}{7} \rightarrow -6 \cdot \frac{-2}{7} \rightarrow \frac{-6 \cdot -2}{1 \cdot 7} \rightarrow \frac{12}{7}\) Notice: \(-6 = \frac{-6}{1}\)

c) \(\frac{x}{5} \cdot \frac{-2}{y} \rightarrow \frac{x}{5} \cdot \frac{-2}{y} \rightarrow \frac{-2x}{5y}\) Make sure and treat numerators and denominators as factors.

d) \(2 \cdot \frac{1}{3} \cdot \frac{5}{7} \rightarrow 2 \cdot \frac{1}{3} \cdot \frac{5}{7} \rightarrow \frac{2 \cdot 5}{3 \cdot 7} \rightarrow \frac{10}{21}\) Notice the factors of \(\frac{1}{3}\) and \(\frac{1}{7}\) represent factors of \(3\) and \(7\) in the denominator.

You Try It! Multiply the following fractional factors together.

a) \(\frac{3}{5} \cdot \frac{2}{5}\)

b) \(-5 \cdot \frac{-3}{17}\)

c) \((-5) \cdot \frac{1}{2} \cdot \frac{-3}{1} \cdot \frac{1}{8}\)

d) \(\frac{2}{7} \cdot \frac{x}{3} \cdot \frac{1}{y}\)
Discussion: When asked to take half of a number, why can you either divide the number by 2 or multiply the number by $\frac{1}{2}$?

Example: Half of 10 is 5 since, 

$$10 \div 2 \rightarrow 10 \left( \frac{1}{2} \right) \rightarrow 5$$

“*When you divide, invert and multiply.*”

Also, notice the division symbol $\div$ even looks like a fraction!

### Dividing Fractional Factors

To divide by a fraction, multiply by the *reciprocal* of the divisor. If $a, b, c,$ and $d$ represent *factors*, and $b, c$ and $d$ are not 0, we have

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

As a matter of fact, any division can be rewritten as a product of the *reciprocal* factor. This is what we will call *fractional factors*.

### Composition of Mathematics:

A *fractional factor* is the divisor of a division or is the factor in the denominator of a fraction.

$$a \div b = a \left( \frac{1}{b} \right) = \frac{a}{b}$$

The fraction $\frac{a}{b}$ has a factor of $(a)$ and *fractional factor* of $\left( \frac{1}{b} \right)$.

#### Example 2:

Divide the following fractional factors.

**a)** $15 \div 3$

Note: $\div 3$ represents a factor of $\left( \frac{1}{3} \right)$.

$15 \left( \frac{1}{3} \right) \rightarrow \frac{15}{3} \rightarrow 5$

**b)** $-3 \div \frac{1}{4}$

Note: $\div \frac{1}{4}$ represents a factor of $(4)$.

$-3 \left( \frac{1}{4} \right) \rightarrow -3 \cdot 4 \rightarrow -12$

**c)** $-\frac{3}{5} \div 2$

Note: $\div 2$ is the same as a factor of $\left( \frac{2}{3} \right)$.

$-\frac{3}{5} \left( \frac{2}{3} \right) \rightarrow -\frac{9}{10}$

**d)** $8x \div \left( y + 1 \right)$

Note: $\div \left( y + 1 \right)$ is the same as a factor of $\left( \frac{1}{y + 1} \right)$.

$8x \left( \frac{1}{y + 1} \right) \rightarrow \frac{8x}{y + 1}$

Note: *A rational expression* is any expression containing a fractional term with a variable fractional factor. Therefore, if a fraction contains a variable in the denominator, we can refer to it as a *rational expression*.

A *Fractional Term* uses a *fraction bar* as a grouping symbol (*factor operator*) and is made up of *Fractional Factors*. The fraction bar is a factor operator and acts like a grouping symbol. From the examples above, the *factors* of $\frac{15}{3}$ are, 15 and $\frac{1}{3}$.

The factors of $\frac{8x}{y + 1}$ are 8, $x$, and $\frac{1}{(y + 1)}$.  

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Composition of Mathematics:

The fraction bar of a fractional term is a factor operator (division) and groups the numerator as a factor and the denominator as a factor.

Example 3: Identify the factors in each fractional term.

a) \( \frac{32x}{28y} \rightarrow (32)(x)\left(\frac{1}{28}\right)\left(\frac{1}{y}\right) \)

The factors in the denominator become individual fractional factors.

b) \( -\frac{4}{5} \rightarrow (1)(-\frac{4}{5}) \)

Notice how the negative sign outside of the fraction (factor operator) becomes a factor of \((-1)\).

Note: We can also write \( -\frac{4}{5} \) as \( -\frac{4}{5} \) with factors of \((-4)(\frac{1}{5})\).

c) \( \frac{x+5}{x-5} \rightarrow (x+5)\left(\frac{1}{x-5}\right) \)

The fraction bar acts as a grouping symbol of factors.

d) \( \frac{3}{5} \div \frac{2}{3} \)

Rewrite the divided factor as its reciprocal before identifying the fractional factors.

\( \frac{3}{5} \left(\frac{3}{2}\right) \rightarrow (-1)\left(\frac{3}{5}\right)\left(\frac{3}{2}\right) \)

e) \( -5 \div 7 \)

Rewrite the divided factor as its reciprocal before identifying the fractional factors.

\( -5 \left(\frac{1}{7}\right) \rightarrow (-5)\left(\frac{1}{7}\right) \)

f) \( (x + 5) \div (x - 5) \)

Rewrite the divided factors as their reciprocal before identifying the fractional factors.

\( (x + 5)\left(\frac{1}{x-5}\right) \)

You Try It! Identify the factors in each term. DO NOT SIMPLIFY

a) \( -\frac{3}{7} \)

b) \( -10 \div 15 \)

c) \( \frac{x}{2y} \)

d) \( \frac{5}{7} \div \left(\frac{10}{35}\right) \)

e) \( \frac{x + 5}{x - 5} \)

f) \( \frac{3x}{x + 3} \)

g) \( \frac{30x(y + 1)}{10x(y + 1)} \)

h) \( \frac{y(y + 7)}{y + 7} \)
Objective 3  Simplifying Fractional Terms

Recall: Dividing a number by itself is equal to 1. Except for 0, since division by 0 is undefined.

Composition of Mathematics: Fractional Factor of 1

We have that, \( a + a = 1 \) or, \( \left( \frac{1}{a} \right) \cdot a = 1 \) where \( a \neq 0 \)

We will refer to \( \frac{a}{a} \) as a fractional factor of 1.

From the definition, multiplying reciprocal factors, \( \left( \frac{1}{a} \right) \), produces a fractional factor of 1.

Example 4: Multiply the reciprocal factors together.

a) \( 5 \left( \frac{1}{5} \right) \rightarrow \frac{5}{5} \rightarrow 1 \)

b) \( - \frac{2}{3} \left( - \frac{3}{2} \right) \rightarrow (-1)(-1) \left( \frac{6}{6} \right) \rightarrow 1 \)

c) \( \frac{2x}{3} \left( \frac{3}{2x} \right) \rightarrow \frac{6x}{6x} \rightarrow 1 \)

d) \( \left( \frac{x + 4}{x + 4} \right) \cdot \left( \frac{1}{x + 4} \right) \rightarrow \frac{(x + 4)}{(x + 4)} \rightarrow 1 \)

Composition of Mathematics: The Fundamental Property of Fractions

\[
\frac{A \cdot C}{B \cdot C} \rightarrow \frac{A}{B} \cdot \frac{C}{C} \rightarrow \frac{A}{B}
\]

Interpretation: A fractional term containing a fractional factor of 1 is equivalent to the term with the fractional factor of 1 removed. This makes sense since multiplying by 1 does not change the value of a number or term.

Remember, we always want to take advantage of the fact that we can multiply factors in any order. This allows us to look for the most convenient and efficient factors to multiply together first! We will use the same approach when multiplying fractional factors together!

When multiplying fractional factors, start with the fractional factors of 1 first!

Composition of Mathematics:

To simplify a fractional term, recognize and replace all fractional factors of 1 with a factor of 1 or “Cancel” fractional factors of 1 keeping track of factors that are left. Then, multiply the factors that are left.

The following example relates the idea of cancelling common factors between the numerator and denominator of a fraction. This is exactly what a fractional factor of 1 is... a fraction of common factors between the numerator and denominator.
**Example 5:** Simplify each of the following fractional terms using the given method:

<table>
<thead>
<tr>
<th>Replacing Fractional Factors of 1 with a factor of 1</th>
<th>Canceling Fractional Factors of 1, keeping track of factors that are left.</th>
</tr>
</thead>
</table>
| 1a) \[
\frac{24}{54} \rightarrow \frac{4(6)}{(9)(6)} \rightarrow \frac{4(1)}{9} \rightarrow \frac{4}{9}
\] | 2a) \[
\frac{24}{54} \rightarrow \frac{4}{9}
\] |
| 1b) \[
\frac{45}{30} \rightarrow \frac{(3)(15)}{(2)(15)} \rightarrow \frac{3(1)}{2} \rightarrow \frac{3}{2}
\] | 2b) \[
\frac{45}{30} \rightarrow \frac{3}{2}
\] |
| 1c) \[
\frac{-7x}{-14xy} \rightarrow \frac{(-7)(x)}{2(-7)(x)(y)} \rightarrow \frac{1}{2} \frac{1}{y} \rightarrow \frac{1}{2y}
\] | 2c) \[
\frac{-7x}{-14xy} \rightarrow \frac{1}{2 \frac{1}{y}} \rightarrow \frac{1}{2y}
\] |

Recall from Example 4, when multiplying fractions, we can group all the factors together in one fractional term. Therefore, multiplying fractions is the same as simplifying a fractional term. Remember, take advantage of the fact that we can multiply factors in any order. Don’t just multiply left to right, look for and cancel fractional factors of 1 first.

**Example 6:** Simplify the following fractional terms.

**a)** \[
\frac{5}{9} \cdot \frac{18}{15} \rightarrow \frac{1}{9} \frac{2}{3}
\] The factor of 9 cancels with the factor of 18 leaving a factor of 2 in the numerator.

The factor of 5 cancels with the factor of 15 leaving a factor of 3 in the denominator.

**b)** \[
\frac{2}{3} \div \frac{5}{4} \rightarrow \frac{2}{3} \cdot \frac{4}{5}
\] Rewrite the divided factor as its reciprocal.

The factors of 3 cancel…fractional factor of 1.

The factor of 2 cancels with the 4 leaving a factor of \(\frac{1}{2}\) in the denominator.

The factor of 5 cancels with the 10 leaving a factor of \(\frac{1}{5}\) in the denominator.

**c)** \[
3a \cdot \frac{(2b)(b)}{5a} \rightarrow \frac{3a}{1} \frac{2b}{10a}
\] Rewrite the divided factor as its reciprocal.

Cancel fractional factors of 1, keeping track of factors that are left.

Then, multiply the factors that are left.

**d)** \[
\frac{x+1}{3(x+1)} \rightarrow \frac{1}{3}
\] The fraction bar acts like a grouping symbol of the grouped factor in the numerator.

**e)** \[
\frac{5a}{a+5} \rightarrow \frac{5a}{a+5}
\] The fraction bar acts like a grouping symbol of the grouped factor in the denominator.

This fractional term is simplified, since there are no common factors between the numerator and denominator to cancel.

**Problem Structure Tip:**

Work fractions horizontally as you simplify to keep track of factors in the numerator and fractional factors in the denominator easier.

**Note:** Multiplying fractions is the same as simplifying a fractional term since, \[
\frac{5}{9} \div \frac{18}{15} \rightarrow \frac{5}{9} \cdot \frac{15}{18}
\]

This allows us to multiply factors in any order.

**Tip:** If you multiply the fractions without canceling first, you would have a hard time simplifying the fraction after.

\[
\frac{5}{9} \div \frac{18}{15} \rightarrow \frac{90}{27} \rightarrow \frac{2}{3}
\]

**Discussion:** How is \(3 \div (2x)\) different from \(3 + 2x\)? What are the fractional factors in each?
You Try It!  Simplify the following terms containing fractional factors.

a) \( \frac{25}{30} \)

b) \( \left( \frac{16}{3} \right) \left( \frac{7}{8} \right) \left( \frac{5}{21} \right) \)

c) \(-9x \div (18xy)\)

d) \( -\frac{22}{3} \div \left( -\frac{11}{12} \right) \left( \frac{5}{8} \right) \)

e) \( \frac{x+7}{5(x+7)} \)

f) \( \frac{4y}{y+4} \)
Sec 1.6 Problem Set

Problem Set Directions:
Section problem sets are used to help practice and master topics covered in each section. Each problem should be copied, completed and organized in a separate homework notebook. This homework notebook will be used as a study guide when studying and reviewing for an exam. For each problem in the problem set:
1) Rewrite the directions for each group of problems.
2) Rewrite the problem
3) Show all problem solving steps with correct problem structure. Use the examples in the section as a guide for how to show work.
4) Circle your answer.

Identify each term as a product of fractional factors. DO NOT SIMPLIFY Follow Example 1

1. \(-\frac{4}{5}\) 2. \(24 + 20\) 3. \(\frac{32x}{28y}\) 4. \(-\frac{3}{8} \left( \frac{-16}{9} \right)\) 5. \(-\frac{5}{7} \left( \frac{10}{35} \right)\)

6. \(\frac{30x(y+1)}{10x(y+1)}\) 7. \(12 \div 5(-15) + 6\) 8. \(\frac{3x}{x+3}\) 9. \((x+5) + (x-5)\) 10. \(\frac{y(y+7)}{y+7}\)

Simplify the following terms containing fractional factors.

11. \(3 \left( \frac{2}{5} \right)\) 12. \(-2 \left( \frac{1}{3} \right) \left( -\frac{2}{5} \right)\) 13. \(\frac{2}{7} \div \frac{3}{y}\) 14. \(3x + y\) 15. \(-\frac{3}{8} \left( \frac{8}{3} \right)\)

16. \(-\frac{5}{7} + \frac{10}{35}\) 17. \(\frac{25}{30}\) 18. \(\frac{18}{-6}\) 19. \(3x + (6x)\) 20. \(2 \times x + 3\) 21. \(\frac{3}{16} \left( \frac{16}{9} \right)\)

22. \(-\frac{5}{7} \div \frac{5}{7}\) 23. \(\left( \frac{16}{3} \right) \div \left( \frac{2}{8} \right) \div \left( \frac{5}{21} \right)\) 24. \(7 \div 10 \left( \frac{20}{21} \right)\) 25. \(\frac{5}{7} \div (-10)\) 26. \(\frac{30x(y+1)}{10x(y+1)}\)

27. \(\frac{1}{3} \left( \frac{6}{5} \right) \div \left( \frac{4}{10} \right)\) 28. \(\frac{0}{3}\) 29. \(\frac{2}{0}\) 30. \(-\frac{2}{3} \div 5\) 31. \(-\frac{2}{3} \div \left( \frac{12}{11} \right) \div \left( \frac{5}{8} \right)\)

32. \(\frac{32x}{28y}\) 33. \(\left( \frac{52}{60} \right) \div \left( \frac{12}{68} \right)\) 34. \(-\frac{3}{5} \div \left( \frac{1}{25} \right) \div (-6)\) 35. \(12 \div 5(15) \div 6\)

36. \(-2 \left( \frac{2}{5} \right) \div \left( \frac{3}{5} \right)\) 37. \(\frac{3x}{y} \div \left( \frac{2y}{5x} \right) \div \left( \frac{y}{10x} \right)\) 38. \(\frac{3x}{5(x+1)} \div \frac{10(x+1)}{9x}\) 39. \(\frac{7(a+2)}{3a} \div \frac{5(a+2)}{6a}\)

40. \(-\frac{2x}{3} \div (-6x)\) 41. \\(\frac{3a}{b} \div \frac{2a}{3} \div \frac{2}{5b}\) 42. \(3 \cdot \frac{1}{x+3} \cdot x\) 43. \(\left( x+5 \right) \left( \frac{1}{x-5} \right)\)

44. \(\frac{y(y+7)}{y+7}\) 45. \(\frac{x+2}{2x(x+2)}\) 46. \(\frac{529}{60} \div \left( \frac{120}{173} \right) \div \left( \frac{173}{529} \right) \div \left( \frac{17}{2} \right)\)
Chapter 1 Practice Exam

Define the **composition** of each expression below. Use a **vertical dotted line** to separate the terms and write the factors of each term using **parentheses ( )** or the **factor operator**. **DO NOT SIMPLIFY.** *(Review Sec 1.3 and 1.4)*

1. \(-2 - 3\sqrt{4}\)
2. \(-4^2 - 3^2\)
3. \(-3|2\sqrt{36}(-4)|^2\)
4. \(-x(x + 3)\)
5. \(-2|-3 + 7| - \sqrt{16} + 2(-2)^2\)

Write a term consisting of the given factors. *(Review Sec 1.3)*

6. \(3, 4\) and \(a\)
7. \(2, x, x, x, x\)

8. Write 42 as a product of primes. *(Review Sec 1.2)*

9. List all the factors of 72. *(Review Sec 1.2)*

10. Find the Greatest Common Factor (GCF) between 28, 56, and 42. *(Review Sec 1.2)*

Evaluate the following terms. *(Review Sec 1.3)*

11. \(-2\sqrt{16}\)
12. \(-(-5 + 7)\sqrt{25 - 16}\)
13. \(-3|2\sqrt{36}(-4)|^2\)

Evaluate the following expressions. *(Review Sec 1.4)*

14. \(5 - 4(3)\)
15. \(5(-2)^2 - 2\sqrt{25} + 20\)
16. \(-4^2 - 3^2\)
17. \(2(3 - 5)^2 - (-2 - 3)^2\)
18. \(-2|3 + 7| - \sqrt{16} + 2(-2)^2\)

Evaluate the following expressions for \(x = -2, y = 4, z = -1\) *(Review Sec 1.5)*

19. \(-4x - y + 3z\)
20. \(-(x + z)^2 + 3\sqrt{2y - z}\)

Simplify the following algebraic expressions. *(Review Sec 1.5)*

21. \(3x - 7 - 5x + 10\)
22. \(7 - 3(4 - a) - (3a - 4)\)

23. Translate the word phrase to an algebraic expression. Let \(x\) represent a number. *(Review Sec 1.5)*
   
a) five less than twice a number \(\rightarrow\)
   
b) the sum of a number and 4, divided by 3 \(\rightarrow\)

24. Translate the algebraic expression to a word phrase. Let \(x\) represent a number and use the words **sum** and **difference** when appropriate. *(Review Sec 1.5)*
   
a) \(-3 + x\) \(\rightarrow\)
   
b) \(3(x - 2)\) \(\rightarrow\)

Identify the factors of each term. **DO NOT SIMPLIFY** *(Review Sec 1.6)*

25. \(-\frac{5x}{8y}\)
26. \(\frac{5}{7} \times \left(\frac{10}{21}\right) \div 3\)
27. \(\frac{3x}{x + 3}\)

Write a fractional term with the given list of factors. *(Review Sec 1.6)*

28. \(-1, 5, \frac{1}{7}, x, \text{ and } \frac{1}{x + 1}\)

Simplify the following terms containing fractional factors. *(Review Sec 1.6)*

29. \(\frac{25}{30}\)
30. \(-\frac{5}{7} \times \left(\frac{10}{21}\right)\)
31. \(\left(\frac{36}{25}\right) \times \left(\frac{10}{9}\right) \times \left(-\frac{5}{2}\right)\)
32. \(\frac{1}{2x} \left(\frac{6y}{5}\right) \div \left(\frac{3y}{20x}\right)\)
33. \(\frac{y - 2}{3(y - 2)}\)