Derivation of the Inegrated Rate Law

Note Title 7/16/2008

In a first order process, how is concentration related to time?

$aA \rightarrow$ products (assume 1st order)

$$\text{Rate} = -\frac{\Delta[A]}{\Delta t} \qquad \text{Rate} = k[A]$$

$$-\frac{d[A]}{dt} = k[A]$$

$$-\frac{d[A]}{[A]} = k\,dt$$

Integrate over time range where time = 0 to time = t.
$t = 0$ to t $\quad [A] = [A]_0$ to $[A]_t$

$$-\int_{[A]_0}^{[A]_t} \frac{d[A]}{[A]} = k\int_0^t dt$$

Recall that:

$$\int \frac{dx}{x} = \ln x$$

$$-\ln[A]\Big|_{[A]_0}^{[A]_t} = kt\Big|_0^t$$

$$\ln\frac{x}{y} = \ln x - \ln y$$

$$-\left(\ln[A]_t - \ln[A]_0\right) = kt - k\cdot 0$$

$$\boxed{\ln\frac{[A]_t}{[A]_0} = -kt}$$

Inegrated Rate Law for a 1st order process