

Integer Exponents and Scientific Notation

Recall the definition of an exponent and the rules for operating with exponents from Beginning Algebra. An **exponent** tells the number of times a base is to be used when multiplying. For example, 4^3 means $(4)(4)(4)$ where 3 is the exponent and 4 is the base.

The following rules apply for all integers m and n and all real numbers a and b

Product Rule for Exponents: $a^m \cdot a^n = a^{m+n}$

Zero Exponent Rule: $a^0 = 1$

Negative Exponent Rule:

a) $a^{-n} = \frac{1}{a^n}$

$$\text{b) } \frac{1}{a^{-n}} = a^n$$

$$\text{c) } \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$$

$$\text{d) } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Quotient Rule for Exponents: $\frac{a^m}{a^n} = a^{m-n}$

CAUTION: Remember subtraction is not commutative. Look at the order of m and n .

Power Rules for Exponents:

$$\text{a) } (a^m)^n = a^{mn}$$

b) $(ab)^m = a^m b^m$

c) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example 1: Write the meaning of the following:

a) x^4

b) $4x$

c) $x^5 + x^2$

Example 2: Simplify each expression so that no negative exponents appear in the final result. Leave the answer in fraction form and reduce if possible.

a) $(-5)^{-3}$

b) $\left(\frac{3}{2}\right)^{-2}$

c) $\frac{p^{-7}}{p^{-8}}$

d) $-2^2(x^3y^5)^0$

e) $(4x^{-2}y^4)^{-2}(2x^5y^{-3})^2$

$$\text{f) } \frac{18r^{-8}s^{-2}t^0}{6r^{-10}s^{-4}t^{-3}}$$

$$\text{g) } \left(\frac{x^{-3}y^{-3}}{x^{-5}y^6} \right)^{-1}$$

Scientific Notation is used to express very large and very small numbers. A number in scientific notation is written in the form $a \times 10^n$ where $1 \leq |a| < 10$ and n is an integer.

To convert to scientific notation:

- 1) Position the decimal point by placing a caret to the right of the first nonzero digit.
- 2) Determine the numeral for the exponent by counting the number of digits from the decimal point to the caret.
- 3) Determine the sign of the exponent. Decide whether multiplying by 10^n should make the result of step 1 larger or smaller. The exponent should be positive to make the result larger; it should be negative to make the result smaller.

To convert from scientific notation if the power is positive move the decimal point to the right the same number of places as the power and if the power is negative move the decimal point to the left the same number of places as the power.

Example 3: Write each number in standard notation:

a) 2.51×10^3

b) -6.8×10^{-4}

Example 4: Write each number in scientific notation:

a) 29,800,000

b) 0.0000000503

Be able to read the scientific notation on your calculator.