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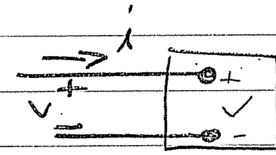
CIRCUITS II

Ch 10 Sinusoidal Steady-State Power Calculations

Instantaneous Power



$p = vi$ amplitude differs

Let $v = V_m \cos(\omega t + \theta_v)$ phase angle differs

$i = I_m \cos(\omega t + \theta_i)$

- For steady-state, the reference time (zero) $(t=0)$ is arbitrary.
- In practice select zero time such that the current is peak

~~$(\omega t + \theta_i) = 0 \Rightarrow \omega t \Rightarrow$ shift by θ_i~~

$i = I_m \cos \omega t$

$v = V_m \cos(\omega t + \theta_v - \theta_i)$

$$\boxed{p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t}$$

Apply trigonometric identities

$$\boxed{p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos 2\omega t - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t}$$

or Let $\underline{P} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \Rightarrow$ average (real) power

units = Watts (W)

$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \Rightarrow$ reactive power

units = volt-amp reactive var (VAR)

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$$\boxed{p = \underline{P} + \underline{P}\cos 2\omega t - Q\sin 2\omega t}$$

$$\underline{P} = \frac{1}{T}\int_{t_0}^{t_0+T} p\,dt$$

Purely Resistive Circuits

Current and voltage responses possess the same phase angle $\theta_v = \theta_i$

$$Q = 0$$

$$\boxed{p = \underline{P} + \underline{P}\cos 2\omega t}$$

Instantaneous real power

reactive power = 0

Purely Inductive Circuits

$$\theta_i = \theta_v - 90^\circ \quad \text{or} \quad \theta_v - \theta_i = 90^\circ$$

$$\underline{P} = 0$$

$$\boxed{p = -Q\sin 2\omega t}$$

average power = 0

Purely Capacitive Circuits

$$\theta_i = \theta_v + 90^\circ \qquad \theta_v - \theta_i = -90^\circ$$

$$\boxed{p = Q\sin 2\omega t}$$

average power = 0

average power => electric energy => thermal energy
reactive power => energy extracted from or stored in electric fields

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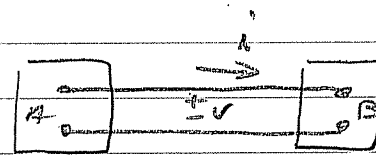
Power Factor

power factor angle = $\theta_v - \theta_i$
- influences the average and reactive power computations

power factor (pf) => $\cos(\theta_v - \theta_i)$
reactive factor (rf) => $\sin(\theta_v - \theta_i)$

Drill Exercise 10.1

Calculate - average & reactive power
- Indicate power flow



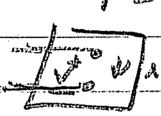
(a) $v = 100\cos(\omega t - 45^\circ)\ V$, $i = 20\cos(\omega t + 15^\circ)\ A$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$= \frac{(100V)(20A)}{2}\cos(-45^\circ - 15^\circ)$$

$$\boxed{P = 500W \quad (p = vi) \quad (A \text{ to } B)}$$

p=vi



$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$Q = \frac{(100V)(20A)}{2}\sin(-45^\circ - 15^\circ)$$

$$\boxed{Q = -866\ VAR \quad (B \text{ to } A)}$$

(b) $v = 100\cos(\omega t - 45^\circ)\ V$, $i = 20\cos(\omega t + 165^\circ)\ A$

$$P = \frac{(100V)(20A)}{2}\cos(-45^\circ - 165^\circ) = -866\ W\ (B \text{ to } A)$$

$$Q = \frac{(100V)(20A)}{2}\sin(-45^\circ - 165^\circ) = 500\ VAR\ (A \text{ to } B)$$

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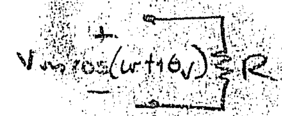


RMS Values and Power Calculations

Use V_{rms} (V_{eff}) and I_{rms} (I_{eff}) instead of V_m and I_m
From before for

$$V_{eff} = \frac{V_m}{\sqrt{2}}, \quad I_{eff} = \frac{I_m}{\sqrt{2}}$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$\boxed{P = V_{eff} I_{eff} \cos(\theta_v - \theta_i)}$$

Similarly

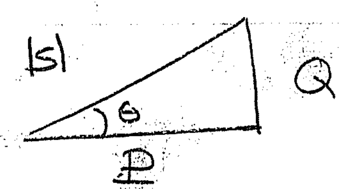
$$\boxed{Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i)}$$

Complex Power

Complex power = (S) sum of the real power and the complex powr

$$S = P + jQ \qquad \text{Units Volt-Amps (VA)}$$

Power Triangle



$$|S| = \sqrt{P^2 + Q^2} \Rightarrow \text{Apparent Power}$$

$$\tan\theta = \frac{Q}{P} = \frac{(V_m I_m/2)\sin(\theta_v - \theta_i)}{(V_m I_m/2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i)$$

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$$S = P + jQ = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}\left(\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right)$$

$$\boxed{S = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)}$$

For rms values

$$\boxed{S = V_{eff} I_{eff} \angle(\theta_v - \theta_i)}$$

$$S = V_{eff} I_{eff} e^{j(\theta_v - \theta_i)}$$

$$= \underbrace{V_{eff} e^{j\theta_v}}_{\mathbf{V}_{eff}} \underbrace{I_{eff} e^{-j\theta_i}}_{\mathbf{I}^*_{eff}}$$

due to minus sign

Note: $\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$$\boxed{S = \mathbf{V}_{eff} \mathbf{I}^*_{eff}}$$

Could do the same for Im and Vm

$$S = \frac{1}{2} V_m I_m e^{j(\theta_v - \theta_i)}$$

$$= \frac{1}{2}\underbrace{\left(V_m e^{j\theta_v}\right)}_{\mathbf{V}}\underbrace{\left(I_m e^{-j\theta_i}\right)}_{\mathbf{I}^*}$$

$$\boxed{S = \frac{1}{2}\mathbf{V}\,\mathbf{I}^*}$$

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Alternate Forms of Complex Power From Phasors

$$V_{eff} = Z I_{eff}$$

$$S = V_{eff} I^*_{eff}$$

$$= Z \underbrace{I_{eff} I^*_{eff}}_{|I_{eff}|^2}$$

$$Z = (R + jX)$$ reactance for inductor or capacitor

$$S = |I_{eff}|^2 (R + jX)$$

$$= \underbrace{|I_{eff}|^2 R}_{P} + j \underbrace{|I_{eff}|^2 X}_{Q}$$

$$\boxed{\begin{aligned} P &= |I_{eff}|^2 R = \tfrac{1}{2} I_m^2 R \\ Q &= |I_{eff}|^2 X = \tfrac{1}{2} I_m^2 X \end{aligned}}$$

Could also use voltages

$$P = \frac{|V_{eff}|^2}{R} \qquad Q = \frac{|V_{eff}|^2}{R}$$

*** Example 10.5

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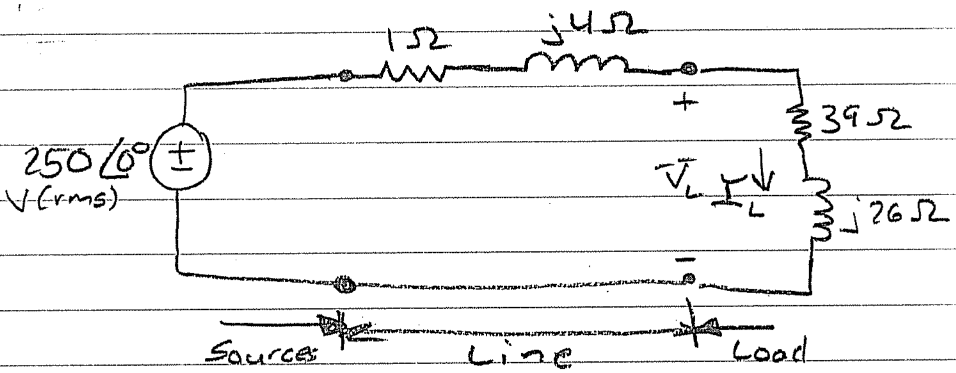
Example 10.5



(a) Find V_L & I_L

$$I_L = \frac{250\angle 0^\circ}{40\,\Omega + j30\,\Omega} = \frac{250\angle 0^\circ}{50\angle 36.87^\circ} = \boxed{5\angle -36.87^\circ \text{ A(rms)}}$$

$-j22$

$$V_L = (I_L)(39 + j26) = (5\angle 36.87^\circ \text{ A})(46.87\angle 33.69^\circ\,\Omega)$$

$$\boxed{V_L = 234.36\angle -3.18^\circ \text{ V (rms)}}$$

(b) Find the average and reactive power delivered to the load

$$P = V_L I_L^*$$

$$I_L = 5\angle -36.87 = 4 - j3 \text{ A (rms)}$$

$$I_L^* = 4 + j3 = 5\angle 36.87^\circ \text{ A(rms)}$$

$$S = [(234.36)\angle -3.18^\circ][5\angle 36.87^\circ]$$

$$S = 1171.8\angle 33.7^\circ \text{ VA}$$

$$\boxed{S = 975 + j650 \text{ VA}}$$

$$\boxed{P = 975 \text{ W}}$$

$$\boxed{Q = 650 \text{ VAR}}$$

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Example 10.5

(c) Calculate the average and reactive power delivered to the ~~load~~

$P = |I_{eff}|^2 R$

$= (5)^2 (1\Omega) = \underline{25 \text{ W}}$

$Q = |I_{eff}|^2 X$

$= (5)^2 (4) = \underline{100 \text{ VAR}}$

$\boxed{S = 25 + j100 \text{ VA}}$

(d) Calculate the average and reactive power delivered by the source

$S = \overset{\text{delivered}}{-}(975 + j650 + 25 + j100)$

$\boxed{S = -(1000 + j750) \text{ VA}}$