CIRCUITS II

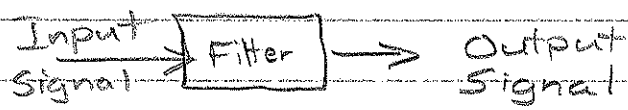
Chapter 14: Frequency Selective Circuits

Sinusoidal Sources => Frequency was held constant

Frequency Response: Analysis of the effect of varying the source frequency

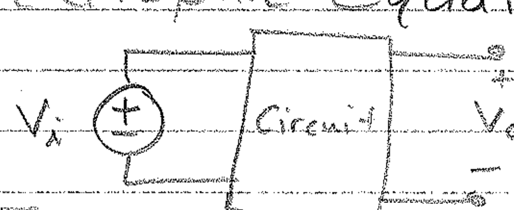
In circuit elements, varying the frequency will affect the impedance of inductors and capacitors.

Frequency Selective Circuits (Filters) => circuits that pass to output only input signals within a certain frequency range



Filters weaken or lessen the effect (attenuate) of signals with frequencies outside a frequency band

- Example (Graphic equalizer)



Transfer Function

From before, Fixed-Frequency sources

Replace with a varying-frequency sourse

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

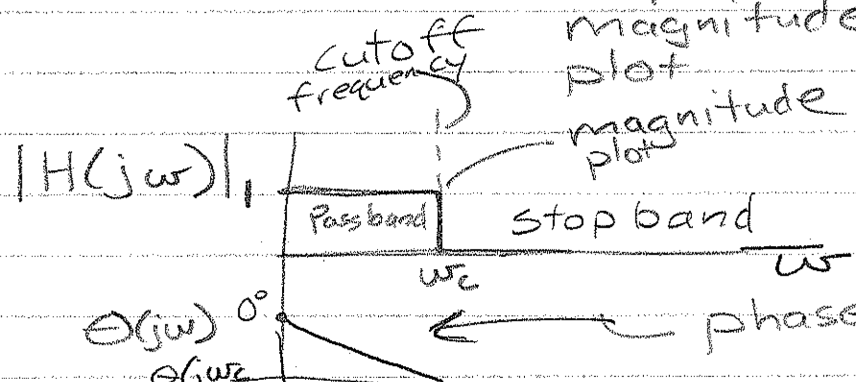
CIRCUITS II

Frequency Bands

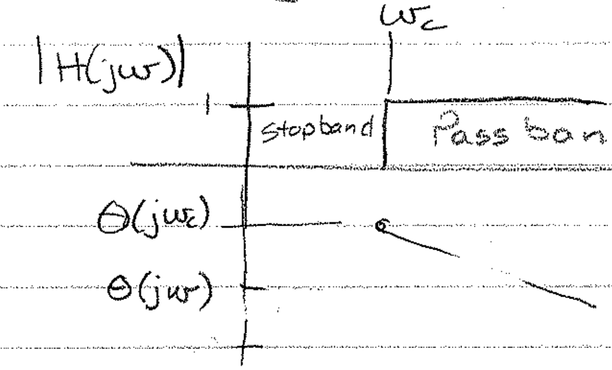
- Pass band => band of frequencies where signals from the input are passed to the output
  Inputs outside this band are attenuated
- Stop Band => Frequencies not in the pass band

Cutoff Frequency =>

Frequency Response Plot - shows how a circuit's transfer function changes as the source frequency changes
=> shows both $\underbrace{\text{magnitude}}_{\text{magnitude plot}}$ and $\underbrace{\text{phase}}_{\text{phase angle plot}}$

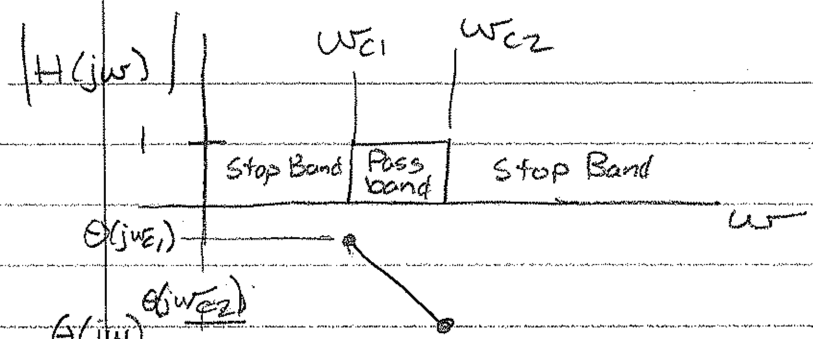


Ideal low-pass filter => passes lower Frequency signals than the cutoff frequency

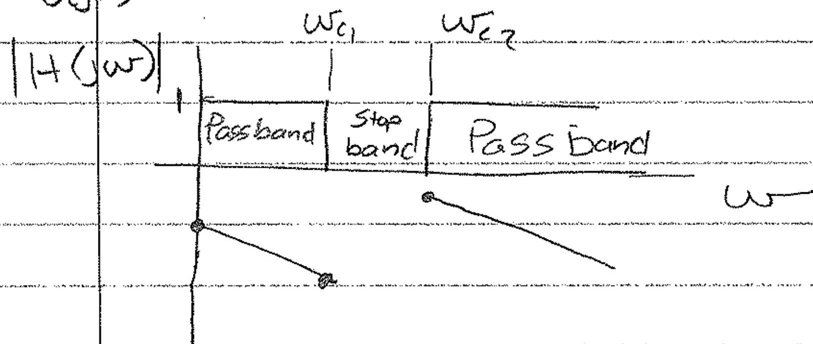


Ideal High-pass filter => passes signals at a higher frequency

CIRCUITS II



Ideal Band-pass filter
=> passes signals within
a frequency band

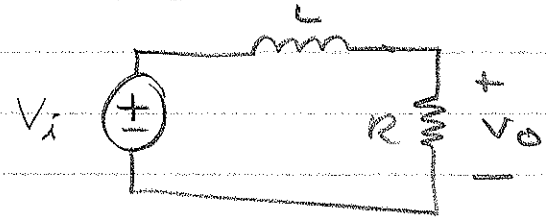


Ideal Band-reject
filter => rejects
signals within a
frequency band

CIRCUITS II

Low-Pass Filters

Series RL Circuit (Qualitative Analysis)



Impedances

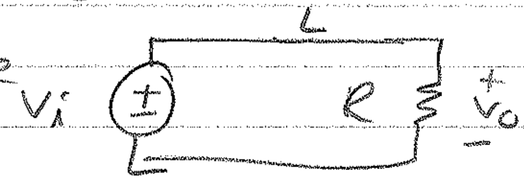
Inductor => $Z = j\omega L$

Resistor => $Z = R$

Low Frequencies => $\omega L << R$

If the inductor impedance is much lower than the resistor impedance, the inductor "effectively" acts as a short circuit

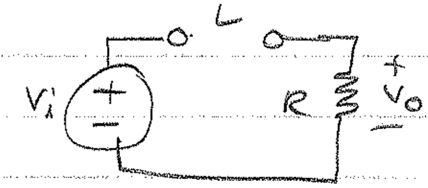
=> output voltage = input voltage



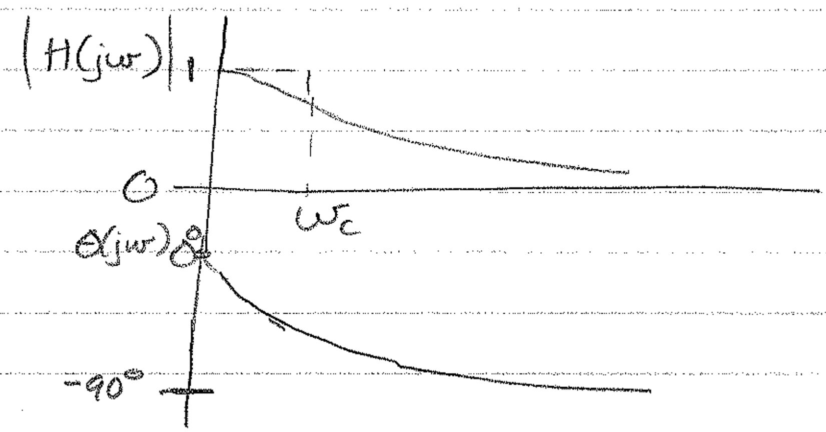
High Frequencies => $\omega L >> R$

Inductor effectively acts as an open circuit

- Magnitude of the output voltage = 0
- phase angle for the output voltage lags the input by 90°



Frequency Response Plot



CIRCUITS II

Cutoff Frequency

In practice, the cutoff frequency is when the transfer function magnitude is decreased by $1/\sqrt{2}$

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

where: H_{max} is the maximum magnitude of the transfer function

Why $1/\sqrt{2}$?

Let V_L be the voltage drop across a load

Average power delivered to the load

$$P = \frac{1}{2}\frac{V_L^2}{R}$$

Maximum Power => when V_L is max

$$P_{max} = \frac{1}{2}\frac{V_{Lmax}^2}{R} \qquad V_{Lmax} = H_{max}|V_i|$$

$$|V_L(j\omega_c)| = \underbrace{|H(j\omega_c)|}_{\frac{1}{\sqrt{2}}H_{max}}|V_i|$$

$$= \frac{1}{\sqrt{2}} H_{max} |V_i|$$

$$= \frac{1}{\sqrt{2}} V_{Lmax}$$

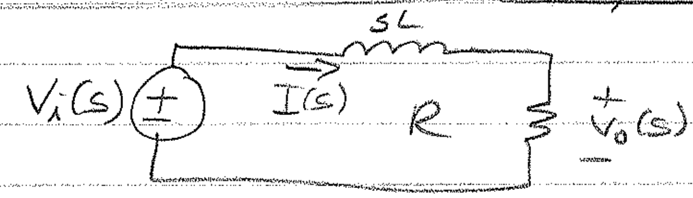
$$P(j\omega_c) = \frac{1}{2}\frac{\left(\frac{1}{\sqrt{2}}V_{Lmax}\right)^2}{R}$$

$$= \frac{1}{2}\underbrace{\left[\frac{\frac{1}{2}V_{Lmax}^2}{R}\right]}_{P_{max}}$$

$$\boxed{P(j\omega_c) = \frac{1}{2}P_{max}} \qquad \omega_c \Rightarrow \text{half-power frequency}$$

CIRCUITS II

Series RL Circuit - Quantitative Analysis



$$-V_i + IsL + IR = 0$$

$$I = \frac{V_i}{sL+R} \qquad V_o = IR = \frac{V_i R}{sL+R}$$

$$V_o = V_i\left(\frac{R/L}{s+R/L}\right)$$

$$\boxed{H(s) = \frac{R/L}{s+R/L}}$$

For the frequency response $\Rightarrow s = j\omega$

$$H(j\omega) = \frac{R/L}{j\omega + R/L}$$

Convert to phasor notation

A.S.A.

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}} \quad \text{(magnitude of } H(j\omega))$$

$$\Theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{(phase angle of } H(j\omega))$$

Cutoff Frequency $|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$

$$H_{max} = |H(j0)| = 1$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}(1) = \frac{R}{\sqrt{\omega_c^2 + (R/L)^2}}$$

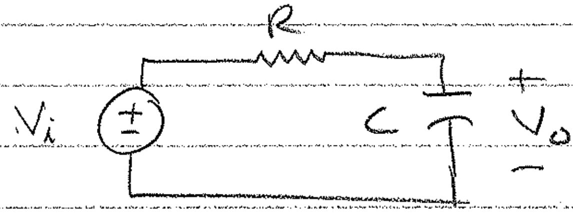
$$\boxed{\omega_c = R/L}$$

CIRCUITS II

Low-Pass Filters

Series RC Circuit

$$Z_C = \frac{-j}{\omega C}$$

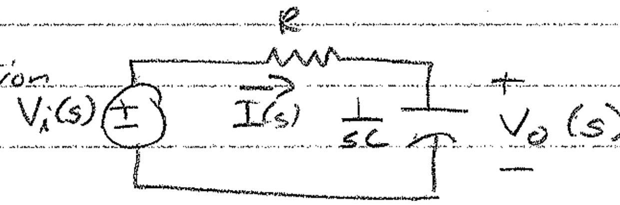


$\omega = 0 \Rightarrow Z_C \Rightarrow \infty$ $\quad V_o = V_i$ (open circuit)

$\omega = \infty \Rightarrow Z_C \Rightarrow 0$ $\quad$ short-circuit $\quad V_o = 0$

Example 14.2

(a) Find the transfer function for $V_o(s)$



KVL

$$-V_i + IR + \frac{I}{sC} = 0$$

$$I = \frac{V_i}{R + \frac{1}{sC}}$$

$$V_o = (I)\left(\frac{1}{sC}\right) = \frac{V_i}{\left(R + \frac{1}{sC}\right)}\left(\frac{1}{sC}\right) = \frac{V_i}{RCs + 1}$$

$$V_o = \frac{V_i\,(1/RC)}{s + 1/RC}$$

$$\boxed{H(s) = \frac{1/RC}{s + 1/RC}}$$

(b) Determine the cutoff frequency, ω_c

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$$

$$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

Low Pass filter $\quad H_{max} = H(j0) = 1$

CIRCUITS II

$$\frac{\frac{1}{RC}}{\sqrt{\omega_c^2 + (1/RC)^2}} = \frac{1}{\sqrt{2}}(1)$$

$$\boxed{\omega_c = 1/RC}$$

(c) Choose values for R and C that will yield a cutoff frequency of 3KHz

$$\omega_c = \frac{1}{RC}$$

$$(3000\,Hz)\left(\frac{2\pi \text{ rad/s}}{1\,Hz}\right) = \frac{1}{RC}$$

$$\frac{1}{RC} = 18849.56$$

Select a value for C => Calculate R
=> less available capacitor values than for resistors or inductors

<u>Let C = 1μF</u>

$$\frac{1}{R(1\times10^{-6}F)} = 18{,}849.56$$

$$\boxed{R = 53.05\ \Omega}$$

In general for RL or RC Circuits

$$\boxed{H(s) = \frac{\omega_c}{s + \omega_c}}$$

<u>RL</u> => $\omega_c = R/L$

<u>RC</u> => $\omega_c = 1/RC$

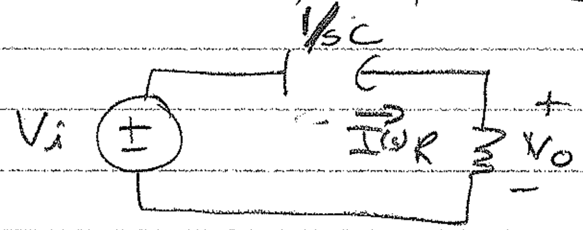
(44)

CIRCUITS II

High-Pass Filters

=> Attenuates Lower ~~f~~frequency input signals

Series RC Circuit



$\omega = 0, \; V_o = 0$

$\omega = \infty, \; V_o = V_i$

Transfer Function

$$-V_i + I\left(\frac{1}{sC}\right) + IR = 0$$

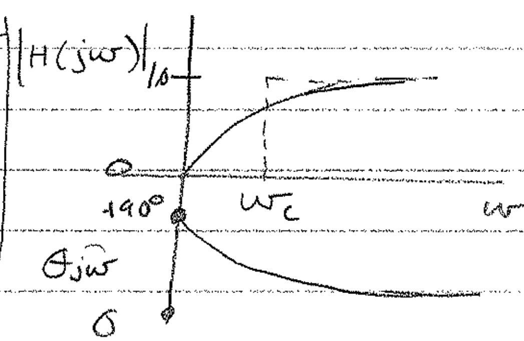
$$I = \frac{V_i}{R + \frac{1}{sC}}$$

$$V_o = IR = \frac{R V_i}{R + \frac{1}{sC}} = \frac{R s V_i}{Rs + \frac{1}{C}} = \frac{s V_i}{s + \frac{1}{RC}}$$

$$H(s) = \frac{s}{s + 1/RC}$$

$$H(j\omega) = \frac{j\omega}{j\omega + 1/RC}$$

$$\boxed{|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + (1/RC)^2}} \qquad \Theta(j\omega) = 90^\circ - \tan^{-1}(\omega RC)}$$



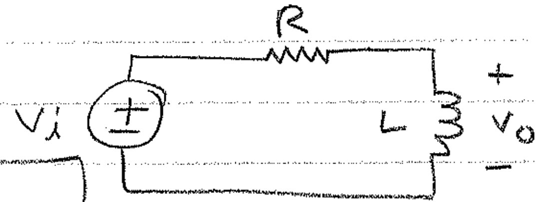
Cutoff Frequency

$$\frac{1}{\sqrt{2}} = \frac{\omega_c}{\sqrt{\omega_c^2 + (1/RC)^2}}$$

$$\boxed{\omega_c = 1/RC}$$

CIRCUITS II

Series RL Circuit



$$\boxed{\begin{aligned} |H(j\omega)| &= \frac{\omega}{\sqrt{\omega^2+(R/L)^2}} \\ |\Theta(j\omega)| &= 90^\circ - \tan^{-1}\left(\frac{\omega L}{R}\right) \end{aligned}}$$

CIRCUITS II

Bandpass Filters

=> Pass Voltages within a frequency band

Parameters

Center Frequency (ω_o) => Frequency where the circuit's transfer function is purely real ($|H(j\omega)|$ is maximum)

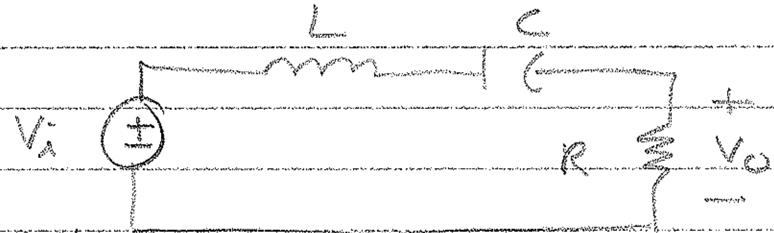
- Also called the resonant frequency

Bandwidth (β) => Width of the pass band

Quality Factor (Q) ratio of the center frequency to the bandwidth

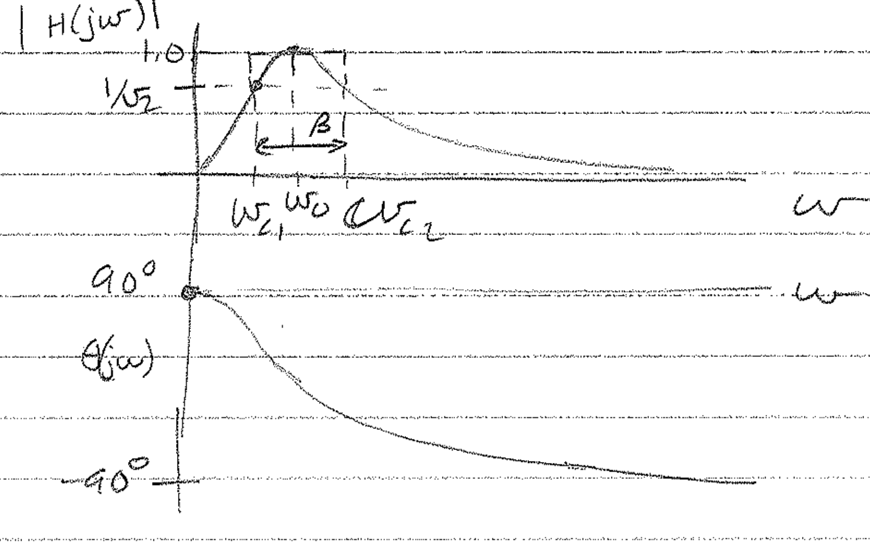
$$\boxed{Q = \frac{\omega_o}{\beta}}$$

Series RLC Circuit



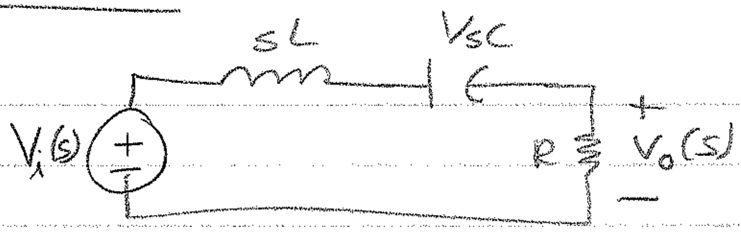
$\omega = 0 \Rightarrow V_o = 0$

$\omega = \infty \Rightarrow V_o = 0$



CIRCUITS II

$$H(s) = \frac{\left(R/L\right)s}{s^2 + \left(R/L\right)s + 1/LC}$$



Substitute $s = j\omega$

$$\boxed{\begin{aligned} \left|H(j\omega)\right| &= \frac{\omega\left(R/L\right)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega R/L\right)^2}} \\ \theta(j\omega) &= 90^\circ - \tan^{-1}\left(\frac{\omega\left(R/L\right)}{\frac{1}{LC} - \omega^2}\right) \end{aligned}}$$

Center Frequency, ω_0

$$Z_C = Z_L = 0$$

$$j\omega_0 L - \frac{j}{\omega_0 C} = 0$$

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC}}}$$

ω_{c1} & ω_{c2}

$$H_{max} = \left|H(j\omega_0)\right| = \frac{\omega_0\left(R/L\right)}{\sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\omega_0 R/L\right)^2}}$$

Substitute $\omega_0 = \sqrt{1/LC}$

$$H_{max} = \underline{1}$$

$$\frac{1}{\sqrt{2}}(1) = \frac{\omega_c\left(R/L\right)}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\omega_c\left(R/L\right)\right)^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\underbrace{\left(\omega_c \frac{L}{R} - \frac{1}{\omega_c RC}\right)^2 + 1}_{2}}}$$

CIRCUITS II

$$\underbrace{\left(\omega_c \frac{L}{R} - \frac{1}{\omega_c RC}\right)}_{\pm 1}^2 + 1 = 2$$

$$\pm 1 = \omega_c \frac{L}{R} - \frac{1}{\omega_c RC}$$

$$\omega_c^2 L \pm \omega_c R - \frac{1}{C} = 0$$

Quadratic Formula => Four answer, only two are positive

$$\boxed{\begin{aligned} \omega_{c1} &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \\ \omega_{c2} &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \end{aligned}}$$

Bandwidth, β

$$\boxed{\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L}}$$

Quality Factor, Q

$$\boxed{Q = \frac{\omega_0}{\beta} = \frac{\sqrt{1/LC}}{R/L} = \sqrt{\frac{L}{CR^2}}}$$

Alternative Forms for Cutoff Frequencies

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 - \omega_0^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$$

$$\omega_{c1} = \omega_0 \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c2} = \omega_0 \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

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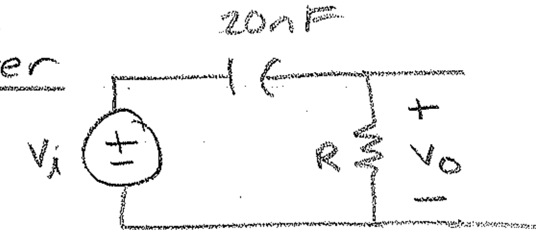
Problem 14.10

High-Pass filter

$f_c = 800\,\text{Hz}$



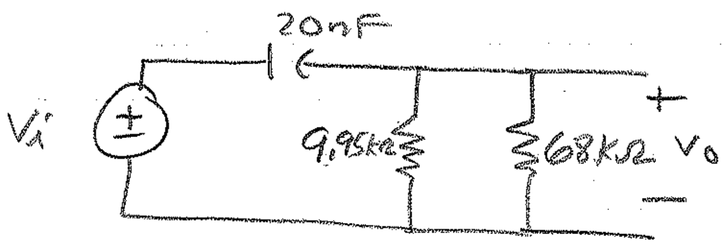
(a) Specify R

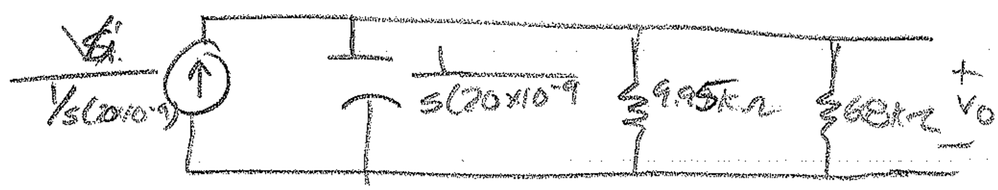
$$\frac{1}{RC} = \omega_c = (2\pi)(800)$$

$$\frac{1}{R(20\times10^{-9})} = (2\pi)(800)$$

$$\boxed{R = 9.95\,\text{K}\Omega}$$

(b) Find ω_c





$$\frac{V_o}{1/(20\times10^{-9}s)} + \frac{V_o}{9.95} + \frac{V_o}{68} = = \underline{V_i(20\times10^{-9}s)}$$

$$V_o = \frac{V_i(20\times10^{-9})s}{\left(20\times10^{-9}s + \frac{1}{9.95\times10^3} + \frac{1}{68\times10^3}\right)}$$

$$H(s) = \frac{s}{s + \frac{1}{20\times10^{-9}}\left(\frac{1}{9.95\times10^3} + \frac{1}{6.8\times10^3}\right)}$$

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{20\times10^{-9}}\left(\frac{1}{9.95\times10^3} + \frac{1}{6.8\times10^3}\right)}$$

$$|H(j\omega)| = \frac{\omega}{\omega^2 + \left[\underbrace{\frac{1}{20\times10^{-9}}\left(\frac{1}{9.95\times10^3} + \frac{1}{68\times10^3}\right)}_{\omega_c}\right]^2}$$

(2)

$$\omega_c = \frac{1}{20\times10^{-9}}\left(\frac{1}{9.95\times10^{3}} + \frac{1}{68\times10^{3}}\right)$$

$$\boxed{\omega_c = 5760 \text{ rad/s}}$$

$$f_c = \frac{5760}{2\pi} = \boxed{917 \text{ Hz}}$$

What if

$$H(s) = \frac{s}{s(4) + 10}$$

$$H(s) = \frac{s/4}{s + \frac{10}{4}}$$

$$H(j\omega) = \frac{1}{4}\left[\frac{j\omega}{j\omega + \frac{10}{4}}\right]$$

$$|H(j\omega)| = \frac{1}{4}\underbrace{\left[\frac{\omega}{\sqrt{\omega^2 + \left(\frac{10}{4}\right)^2}}\right]}_{\text{Max value} = 1}$$

$$H_{max} = \frac{1}{4} \qquad \omega_c = \frac{10}{4}$$

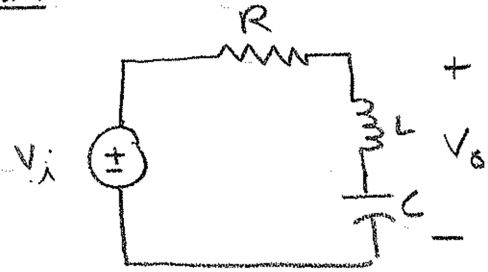
$$H_{max} \leq 1$$

Bandreject Filters

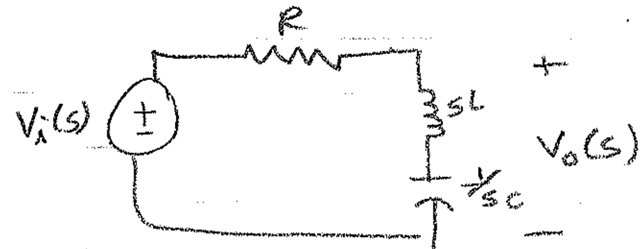
Series RLC Circuit

$\omega = 0 \Rightarrow V_o = V_i$

$\omega = \infty \Rightarrow V_o = V_i$

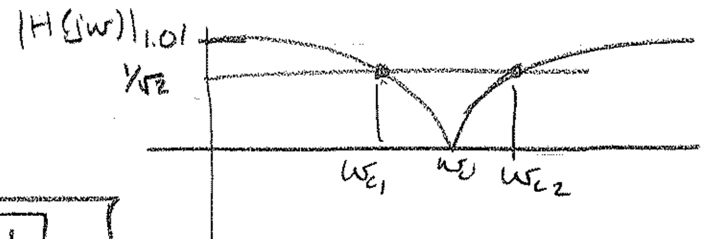


$$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + 1/LC}$$



$$|H(j\omega)| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega R}{L}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega R/L}{\frac{1}{LC} - \omega^2}\right)$$



$$\boxed{\omega_o = \sqrt{\frac{1}{LC}}}$$

$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\beta = \frac{R}{L}$$

$$Q = \sqrt{\frac{L}{R^2 C}}$$

Bode Diagrams

- Efficient method used for generating and plotting amplitude and phase data.

Start with $H(s)$

$$H(s) = \frac{K(s+z_1)}{s(s+p_1)}$$

zeros

constants

poles

$$H(j\omega) = \frac{K(j\omega + z_1)}{j\omega(j\omega + p_1)}$$

Factor out zeros and poles

$$H(j\omega) = \frac{K z_1 (1 + j\omega/z_1)}{p_1 (j\omega)(1 + j\omega/p_1)}$$

(standard form)

Let $K_0 = \frac{K z_1}{p_1}$

$$H(j\omega) = \frac{K_0 (1 + j\omega/z_1)}{(j\omega)(1 + j\omega/p_1)}$$

$$= \frac{K_0 |1 + j\omega/z_1| \angle \psi_1}{(|\omega| \angle 90^\circ)(|1 + j\omega/p_1| \angle \beta_1)}$$

$$= \frac{K_0 |1 + j\omega/z_1|}{(|\omega|)(|1 + j\omega/p_1|)} \angle (\psi_1 - 90^\circ - \beta_1)$$

CIRCUITS II

$$\boxed{|H(j\omega)| = \frac{K_0 \, |1 + j\omega/z_1|}{|\omega| \, |1 + j\omega/p_1|} \qquad \theta(\omega) = \psi_1 - 90^\circ - \beta_1}$$

$$\tan \psi_1 = \frac{\omega/z_1}{1} \Rightarrow \boxed{\psi_1 = \tan^{-1}(\omega/z_1)}$$

$$\tan \beta_1 = \frac{\omega/p_1}{1} \Rightarrow \boxed{\beta_1 = \tan^{-1}(\omega/p_1)}$$

Straight Line Plots

Express the amplitude of $H(j\omega)$ in terms of a logarithmic value: the decibel (dB)

$$A_{dB} = 20 \log_{10} |H(j\omega)|$$

$$\boxed{\begin{aligned} A_{dB} &= 20 \log_{10}\left(\frac{K_0 \, |1 + j\omega/z_1|}{\omega \, |1 + j\omega/p_1|}\right) \\ &= 20 \log_{10}(K_0) + 20 \log_{10}|1 + j\omega/z_1| \\ &\quad - 20 \log_{10}(\omega) - 20 \log_{10}|1 + j\omega/p_1| \end{aligned}}$$

- Plot each term separately, and add graphically to get A_{dB}
- Approximate as straight lines

CIRCUITS II

$20\log_{10}(K_0) =$ constant

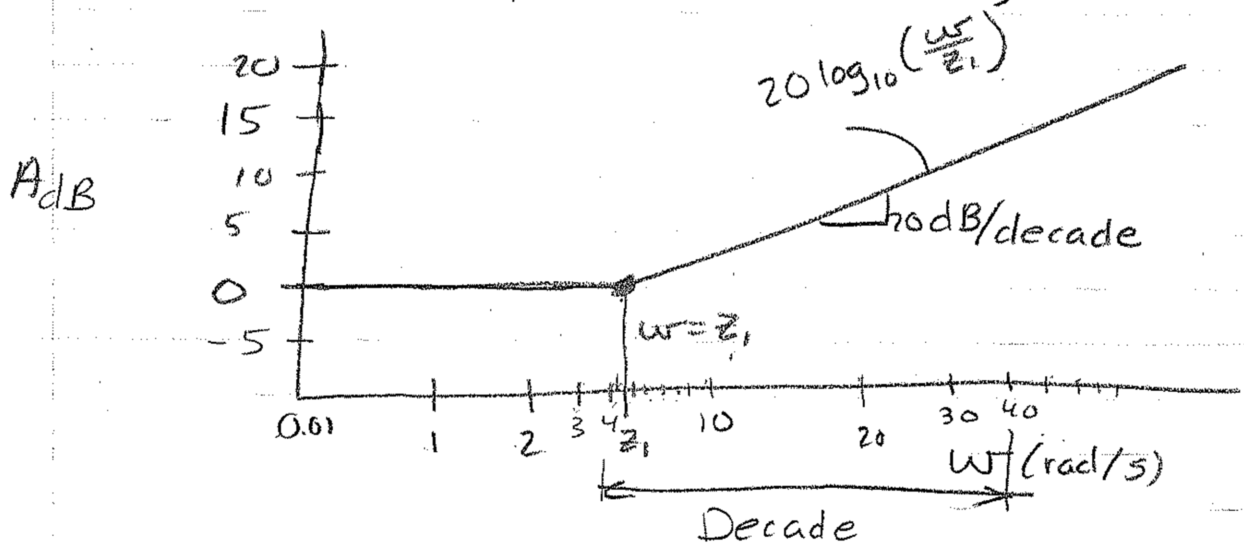
$20\log_{10}|1 + j\omega/z_1|$

Small values of ω

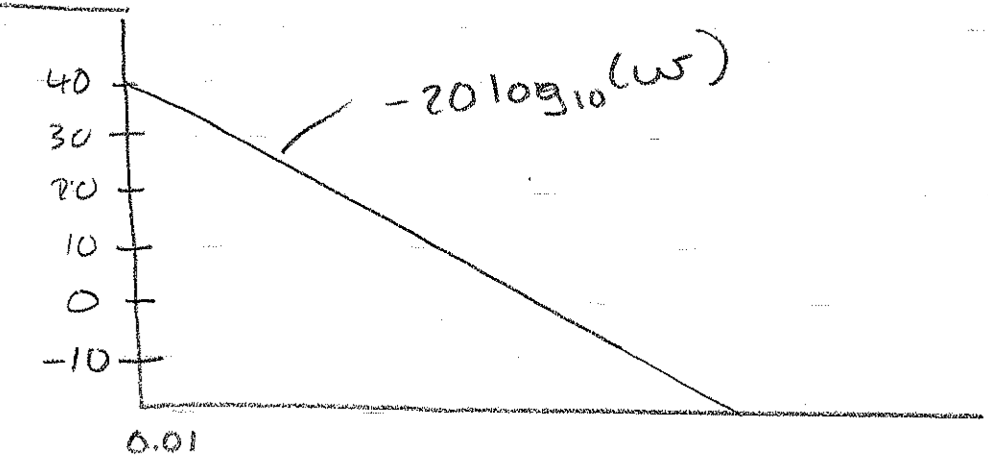
As $\omega \to 0$, $20\log_{10}\left|1+\frac{j\omega}{z_1}\right| \to 0$

Large values of ω

As $\omega \to \infty$; $20\log_{10}(\omega/z_1)$



$-20\log(\omega)$



$-20\log_{10}|1 + j\omega/P_1|$