# CIRCUITS II

## Chapter 15: Active Filter Circuits

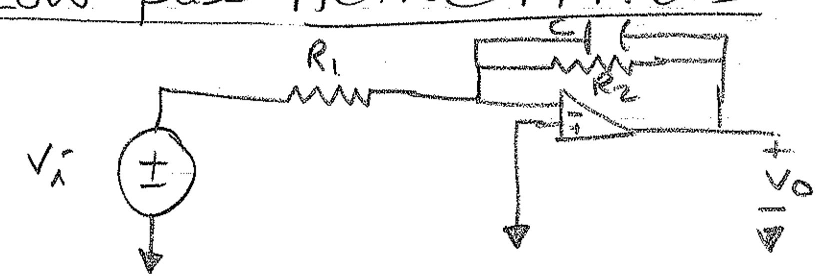
Active Circuits ⇒ Incorporate OP-Amps

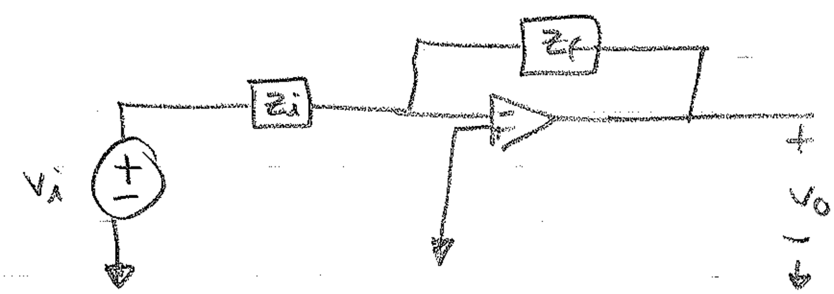
Advantages:

- Produce Bandpass and Bandreject Filters without Inductors
- Inductors are large, heavy, & costly, and can produce electromagnetic field effects

⇒ Assume ideal op-amp

## Low-pass Active Filters




Inverting Op-Amp Circuit

$$V_o = -\frac{Z_f}{Z_i} V_i$$

In the s-domain

$$Z_f = \left(\frac{1}{R_2} + \frac{1}{1/sC}\right)^{-1} = \left(\frac{1}{R_2} + sC\right)^{-1}$$

$$= \left(\frac{sCR_2 + 1}{R_2}\right)^{-1} = \frac{R_2}{sCR_2+1} = \frac{1/C}{s + 1/R_2C}$$

$Z_i = R_1$

$$H(s) = -\frac{1/C}{R_1(s + 1/R_2C)} = -\frac{R_2}{R_1}\left(\frac{1/R_2C}{s + 1/R_2C}\right)$$

# CIRCUITS II

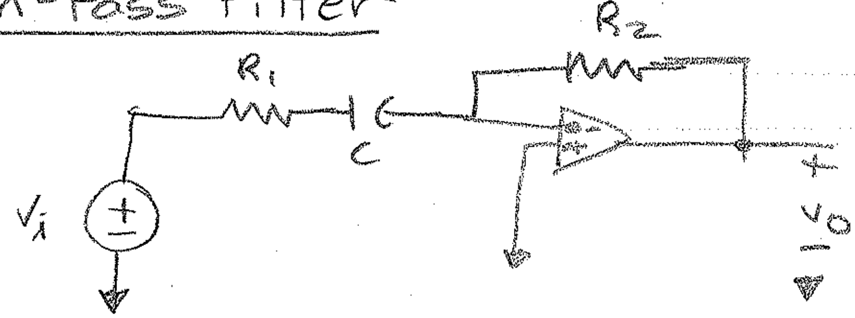
Let $K = \frac{R_2}{R_1}$ & $\omega_c = \frac{1}{R_2 C}$

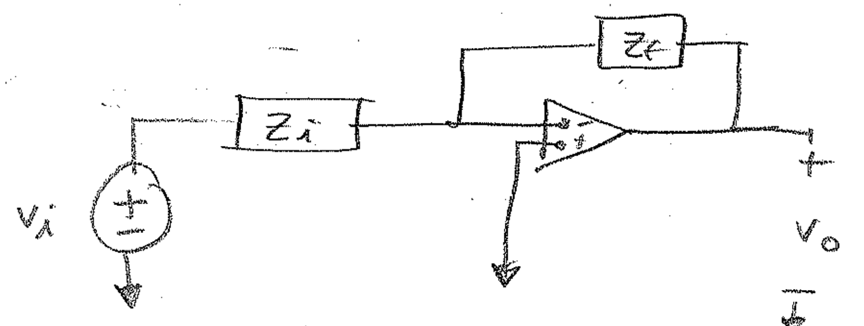
$$\boxed{H(s) = -K \frac{\omega_c}{s+\omega_c}}$$

Same form as for general low-pass filters except for K

K => Gain
=> set by $\frac{R_2}{R_1}$

## High-Pass Filter




$$H(s) = \frac{-Z_f}{Z_i} = \frac{-R_2}{R_1 + 1/sC}$$

$$= \frac{-R_2 s}{sR_1 + 1/C} = \frac{-(R_2/R_1)\, s}{s + 1/R_1 C}$$

$$H(s) = -\left(\frac{R_2}{R_1}\right) \frac{s}{s + 1/R_1 C}$$

# CIRCUITS II

Lek $K = \frac{R_2}{R_1}$, $\omega = 1/R_1C$

$$\boxed{H(s) = -K \frac{s}{s+\omega_c}}$$

Same general form as for high-pass filters.

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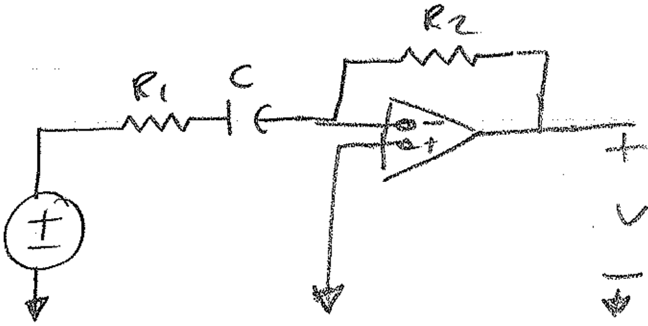
## Drill Exercise 15.1




$R_1 = 1\ \Omega$

**Determine**

$R_2$ & $C$ such that
the passband gain = 1 and $\omega_c$ = 1 rad/s

$$\omega_c = \frac{1}{R_1 C} = 1\text{ rad/s} = \frac{1}{(1\Omega)C}$$

$$\boxed{C = 1F}$$

$$K = \frac{R_2}{R_1}$$

$$1 = \frac{R_2}{1\Omega} \Rightarrow \boxed{R_2 = 1\Omega}$$

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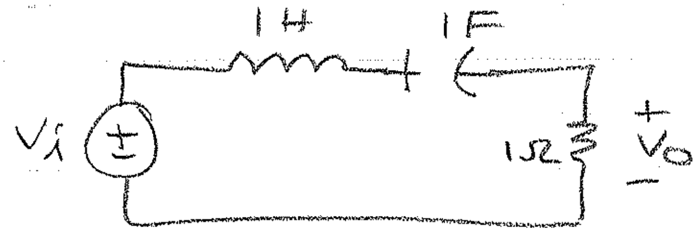
## Examp 15.3




$\omega_o = \frac{1}{\sqrt{LC}} = 1 \text{ rad/s}$

$\beta = R/L = 1 \text{ rad/s}$

$Q = 1$

Use scaling to find R' & L' to yied Q=1 but $f_o = 500\text{Hz}$ with $C = 2\mu F$

$$K_f = \frac{\omega_o'}{\omega_o} = \frac{2\pi(500\text{Hz})}{1 \text{ rad/s}} = \underline{3141.59}$$

$$C' = \frac{1}{K_m K_f} C$$

$$K_m = \frac{1}{K_f}\frac{C}{C'} = \frac{1}{3141.59}\left(\frac{1F}{3141.59(2\times10^{-6}F)}\right) = 159.155$$

$$R' = K_m R = (159.155)(1\Omega) = \underline{159.155\,\Omega}$$

$$L' = \frac{K_m}{K_f}(L) = \frac{159.155}{3141.59}(1H) = \underline{50.66mH}$$

$$\omega_o' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(50.66\times10^{-3}H)(2\times10^{-6}F)}} = 3141.61 \text{ rad/s} = 1\text{Hz}$$

$$\beta = \frac{R'}{L'} = \frac{159.155\,\Omega}{50.66\times10^{-3}} = 3141.6 \text{ rad/s} = 1Hz$$

$$\boxed{Q=1}$$

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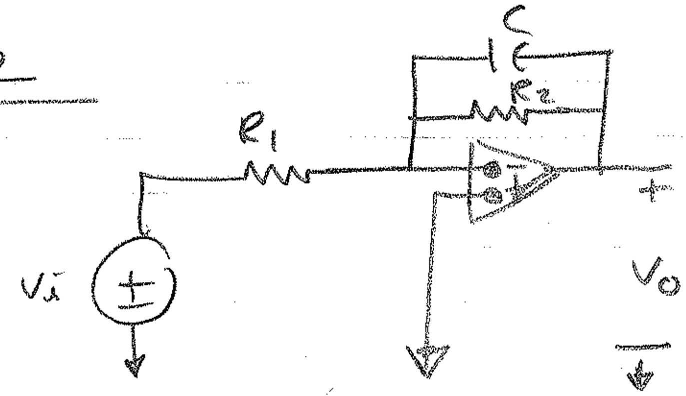
# Drill Exercise 15.2




Low Pass Filter

$$H(s) = \frac{-20{,}000}{s+5000}$$

$C = 5\mu F$

Determine the resistor values

$$\omega_c = 5000 \text{ rad/s} = \frac{1}{R_2 C}$$

$$5000 = \frac{1}{R_2(5\times10^{-6})}$$

$$\boxed{R_2 = 40\,\Omega}$$

$$K = \frac{20{,}000}{5000} = 4$$

$$4 = \frac{R_2}{R_1} = \frac{40\,\Omega}{R_1}$$

$$\boxed{R_1 = 10\,\Omega}$$

## Scaling

Working with values such as $1\Omega$, $1H$, $1F$ is convenient => but unrealistics

- Transform convenient values to realistic ones using scaling

### Magnitude scaling

Multiply the impedance at a given fequency of each element by a scaling factor, $K_m$ or $1/K_m$

$$R' = K_m R, \quad L' = K_m L, \quad C' = C/K_m$$

### Frequency Scaling

- Change the circuit parameters so that the impedances at the new frequency are the same as ~~at~~ the original frequency

$$R' = R, \quad L' = \frac{L}{K_f}, \quad C' = \frac{C}{K_f}$$

Can also scale magnitude and frequency simultaneously

$$\boxed{\begin{aligned} R' &= K_m R \\ L' &= \frac{K_m}{K_f} L \\ C' &= \frac{1}{K_m K_f} C \end{aligned}}$$

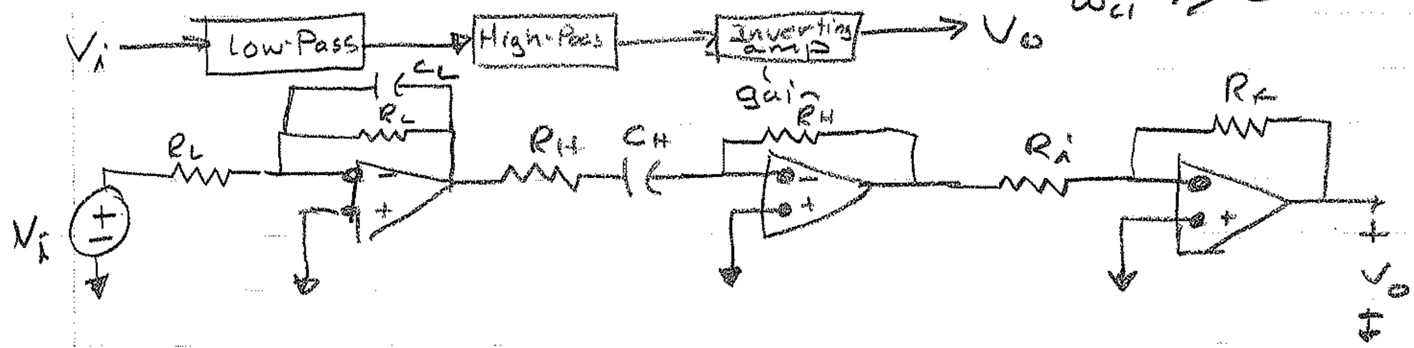
Example 15.3

# CIRCUITS II

## Bandpass Filters

Cascade Three Filters => broadband filter
- passes a wideband of frequency

=> $\frac{\omega_{c2}}{\omega_{c1}} \geq 2$




$$H(s) = \frac{V_o}{V_i}$$

$$= \underbrace{\left(\frac{-\omega_{c2}}{s+\omega_{c2}}\right)}_{\substack{\text{Low-Pass} \\ \omega_{c2} = \frac{1}{R_L C_L}}} \underbrace{\left(\frac{-s}{s+\omega_{c1}}\right)}_{\substack{\text{High-Pass} \\ \omega_{c1} = \frac{1}{R_H C_H}}} \left(-\underbrace{\frac{R_f}{R_i}}_{K}\right)$$

$$H(s) = \frac{-K\omega_{c2}s}{(s+\omega_{c1})(s+\omega_{c2})}$$

$$= \frac{-K\omega_{c2}s}{s^2 + (\omega_{c1}+\omega_{c2})s + \omega_{c1}\omega_{c2}}$$

From Before,

For bandpass filters

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$$

doesn't work

If $\omega_{c2} \gg \omega_{c1}$

$$\Rightarrow \omega_{c1} + \omega_{c2} \approx \omega_{c2}$$

# CIRCUITS II

## Example 15.5

Design a bandpass filter for a graphic equilizer to provide an amplification of 2 within the band of frequencies between 100 and 10,000 Hz. ( Use 0.2 $\mu F$ capacitors)

$$\frac{\omega_{c2}}{\omega_{c1}} = \frac{10{,}000}{100} \Rightarrow 100 \qquad \omega_{c2} >> \omega_{c1}$$

$$\omega_{c2} = \frac{1}{R_L C_L}$$

$$(10{,}000)(2\pi) = \frac{1}{R_L(0.2\times10^{-6}F)}$$

$$\boxed{R_L = 80\Omega}$$

$$\omega_{c1} = \frac{1}{R_H C_H}$$

$$(100)(2\pi) = \frac{1}{R_H(0.2\times10^{-6}F)}$$

$$\boxed{R_H = 7958\Omega}$$

$$K = \frac{R_f}{R_i}$$

$$2 = \frac{R_f}{R_i}$$

Choose $\boxed{\begin{array}{l} R_i = 1000\,\Omega \\ R_f = 2000\,\Omega \end{array}}$

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# CIRCUITS II

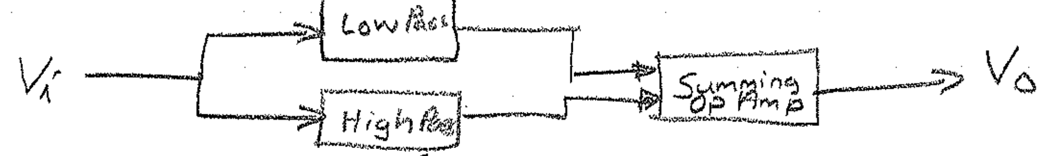
$$H(s) = \frac{-K\omega_{c2}s}{s^2 + \omega_{c2}s + \omega_{c1}\omega_{c2}}$$

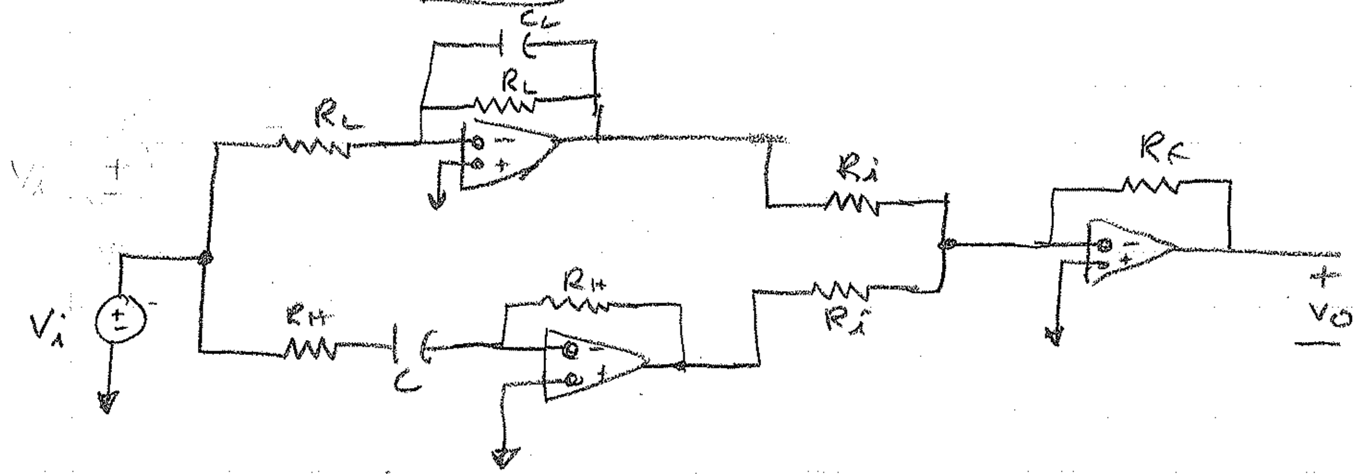
$$\boxed{\begin{aligned}\omega_{c2} &= \frac{1}{R_L C_L} \\ \omega_{c1} &= \frac{1}{R_H C_H}\end{aligned}}$$

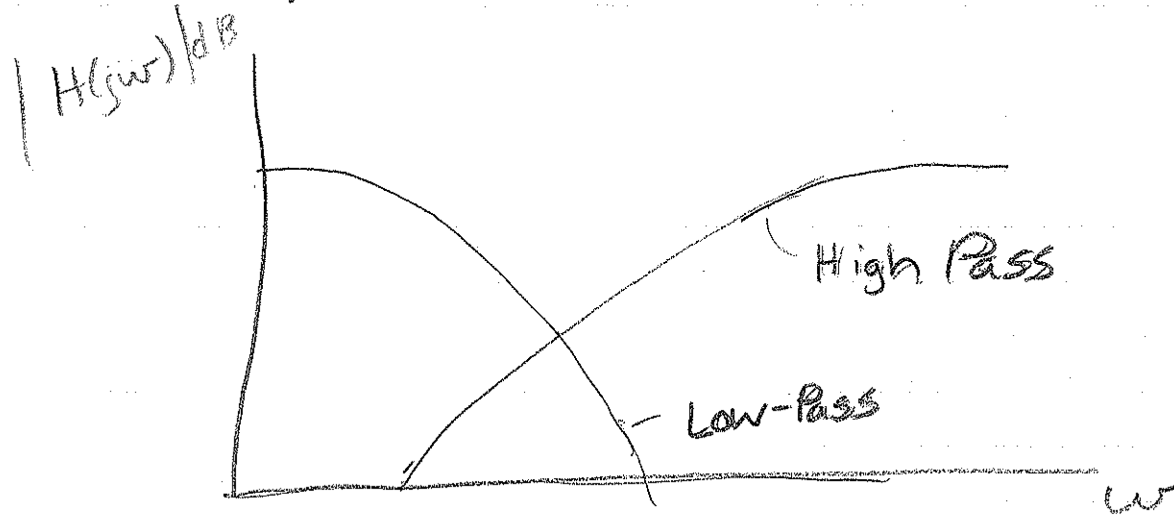
$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

## Band reject Filters

Low-Pass & High-Pass Filters are in parallel



# CIRCUITS II

$$H(s) = -\left(\frac{R_f}{R_i}\right)\left[\frac{-\omega_{c1}}{s+\omega_{c1}} + \frac{-s}{s+\omega_{c2}}\right]$$

$$= \frac{R_f}{R_i}\left(\frac{\omega_{c1}(s+\omega_{c2}) + s(s+\omega_{c1})}{(s+\omega_{c1})(s+\omega_{c2})}\right)$$

$$= \underbrace{\frac{R_f}{R_i}}_{K}\left(\frac{s^2 + 2\omega_{c1}s + \omega_{c1}\omega_{c2}}{(s+\omega_{c1})(s+\omega_{c2})}\right)$$

If $\omega_{c2} \gg \omega_{c1}$

$$\boxed{K = \frac{R_f}{R_i} \qquad \omega_{c1} = \frac{1}{R_L C_L} \qquad \omega_{c2} = \frac{1}{R_H C_H}}$$

Drill Exercise 15.5

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## Drill Exercise 15.5

-Design a parallel bandreject filter

$\omega_0 = 1000 \text{ rad/s}$

$\beta = 4000 \text{ rad/s}$

passband gain = 6

$C = 0.2 \mu F$

$$\beta = \omega_{c2} - \omega_{c1} = 4000$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 1000$$

$$\omega_{c1}\omega_{c2} = 1000^2$$

$$\omega_{c2} = \frac{1\times10^6}{\omega_c}$$

$$\frac{1\times10^6}{\omega_{c1}} - \omega_{c1} = 4000$$

$$-\omega_{c1}^2 - 4000\,\omega_{c1} + 1\times10^6 = 0$$

$$\omega_{c1}^2 + 4000\,\omega_{c1} - 1\times10^6 = 0$$

$$\omega_{c1} = \frac{-4000 \pm \sqrt{(4000)^2 + 4(1\times10^6)}}{2}$$

$$\omega_{c1} = -2000 \pm 2236$$

$$\omega_{c1} = -\cancel{4236},\ +236 \text{ rad/s}$$

$$\omega_{c2} = 4000 + 236 = 4236 \text{ rad/s}$$

$$\frac{\omega_{c2}}{\omega_{c1}} = \frac{4236 \text{ rad/s}}{236 \text{ rad/s}} = 18$$

(2)

## Drill Exercise 15.5

$$\omega_{c2} = \frac{1}{R_H C_H}$$

$$4236 \text{ rad/s} = \frac{1}{R_H(0.2\times10^{-6} F)}$$

$$\boxed{R_H = 1180\ \Omega}$$

$$\omega_{c1} = \frac{1}{R_L C_L}$$

$$236 = \frac{1}{R_L(0.2\times10^{-6} F)}$$

$$\boxed{R_L = 21{,}186\ \Omega}$$

$$\frac{R_f}{R_i} = 6$$

Let $R_i = 1000\ \Omega$
$R_f = 6000\ \Omega$