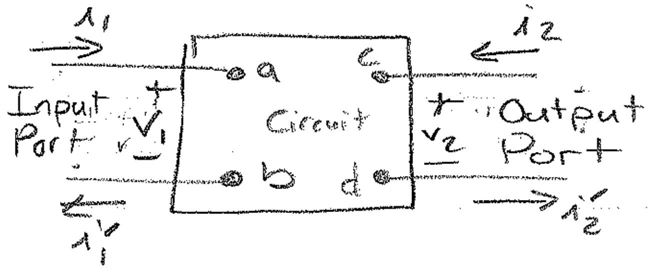
CIRCUITS II

Chapter 18: Two-Port Circuits

- Previously, focused on the behavior of a circuit at a single pair of terminals

- Also can look at two pairs of terminals
Each pair of terminals => port
- Signal fed into a pair of terminals (Input Port) and is then extracted from a a second pair of terminals (output port)
- Can also be More than two ports



Two-port building block

Restrictions

1) No energy can be stored in the circuit

2) No independent sources with the circuit (Can be dependent sources)

3) Current into the port must equal the current out of the port
$i_1 = i_1'$, $i_2 = i_2'$

4) All external connections must be made to either the input or output ports (no connections between ports) i.e. a - c, b-c, a-d)

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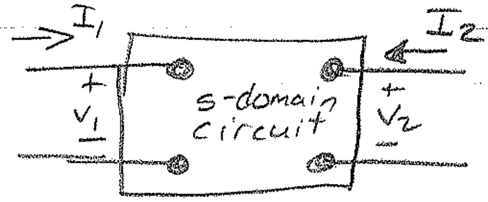
Terminal Equations



- Four terminal variables => I_1, V_1, I_2, V_2
  => Two are independents
- If we know two variables, can solve for the other two using two simultaneous equations.

Six Different Ways

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$
I_1, I_2 known

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$
V_1, V_2 known

$$V_1 = a_{11} V_2 - a_{12} I_2$$
$$I_1 = a_{21} V_2 - a_{22} I_2$$
V_2, I_2 known

$$V_2 = b_{11} V_1 - b_{12} I_1$$
$$I_2 = b_{21} V_1 - b_{22} I_1$$
V_1, I_1 known

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$
I_1, V_2 known

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$
V_1, I_2 known

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Coefficients on the right-hand side
=> parameters of the two-port circuit

z parameters, a parameters, b parameters, h parameters, and g parameters

Two-port Parameters

Can be determined by computation or measurement

z parameters

Two port equations

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$I_2 = 0$

$$\boxed{\begin{aligned} z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0} \\ z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0} \\ z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{aligned}}$$

$I_1 = 0$

z_{11} => impedance seen looking into port 1 when port 2 is open

z_{12} => transfer impedance => ratio of port 1 voltage to port 2 current when port 1 is open

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z_{21} => transfer impedance => ratio of port 2 voltage to port 1 current when port 2 is open

z_{22} => impedance seen looking into port 2 when port 1 is open

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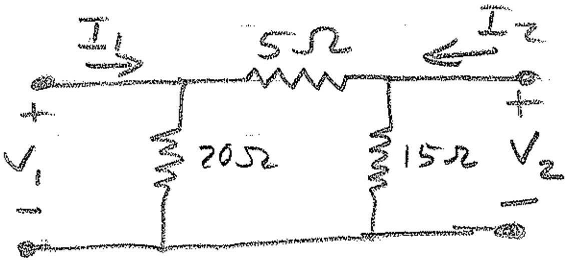
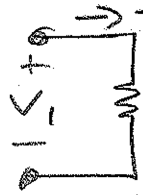
Example 18.1

Determine:
The z-parameter



z_{11} => resistance seen looking into port 1 when port 2 is open, $z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$



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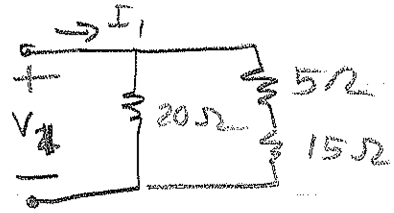
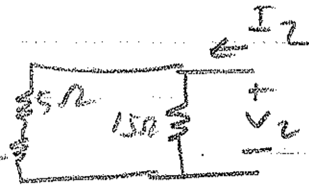


$$z_{11} = \left(\frac{1}{20\Omega} + \frac{1}{5\Omega+15\Omega}\right)^{-1}$$

$$z_{11} = \left(\frac{1}{20\Omega} + \frac{1}{20\Omega}\right)^{-1} = \left(\frac{20+20}{(20)(20)}\right)^{-1}$$

$$= \frac{(20)(20)}{20+20} = \boxed{10\Omega}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \left(\frac{1}{15\Omega} + \frac{1}{20\Omega+5\Omega}\right)^{-1}$$



$$= \left(\frac{25+15}{(15)(25)}\right)^{-1} = \frac{(15)(25)}{25+15}$$

$$\boxed{z_{22} = 9.375\Omega}$$

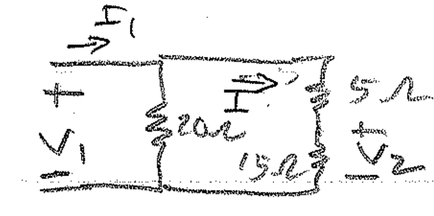
$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Need V_2 & I_1 in terms of a single variable when $I_2 = 0$

$$I_1\Big|_{I_2=0} = \frac{V_1}{z_{11}} = \frac{V_1}{10}$$

Current Divider Circuit

$$I = \frac{20\Omega}{(15\Omega+5\Omega)+20\Omega} = 0.5\, I_1$$



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$$V_2 = (15\Omega)I = 7.5 I_1$$

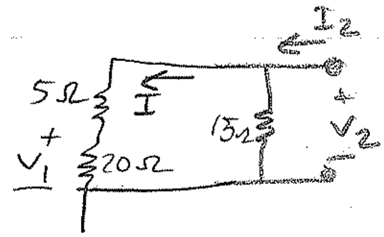
$$V_2 = 7.5\left(\frac{V_1}{10}\right) = 0.75 V_1 \quad \underline{\text{When } I_2 = 0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.75 V_1}{V_1/10} = \boxed{7.5\,\Omega}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

Find V_1 & I_2 in terms of V_2 when $I_1 = 0$

$$I_2 = \frac{V_2}{z_{22}} = \frac{V_2}{9.375}$$



$$I = \frac{15\Omega}{(15\Omega + 5\Omega + 20\Omega)} I_2$$

$$I = 0.375 I_2$$

$$V_1 = 20 I = 7.5 I_2 = 7.5\left(\frac{V_2}{9.375}\right) = 0.8 V_2$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8 V_2}{V_2/9.375} = 7.5\,\Omega$$

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Parameter Conversion

Use matrix Form

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Cramer's Method

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22} I_1 - y_{12} I_2}{\Delta y}$$

$$V_1 = \underbrace{\frac{y_{22}}{\Delta y}}_{z_{11}} I_1 - \underbrace{\frac{y_{12}}{\Delta y}}_{z_{12}} I_2$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{11} I_2 - y_{21} I_1}{\Delta y}$$

$$V_2 = \underbrace{\frac{-y_{21}}{\Delta y}}_{z_{21}} I_1 + \underbrace{\frac{y_{11}}{\Delta y}}_{z_{22}} I_2$$

$$\boxed{z_{11} = \frac{y_{22}}{\Delta y}, \quad z_{12} = \frac{-y_{12}}{\Delta y}, \quad z_{21} = \frac{-y_{21}}{\Delta y}, \quad z_{22} = \frac{y_{11}}{\Delta y}}$$

Parameter Conversion Table => Table 18.1