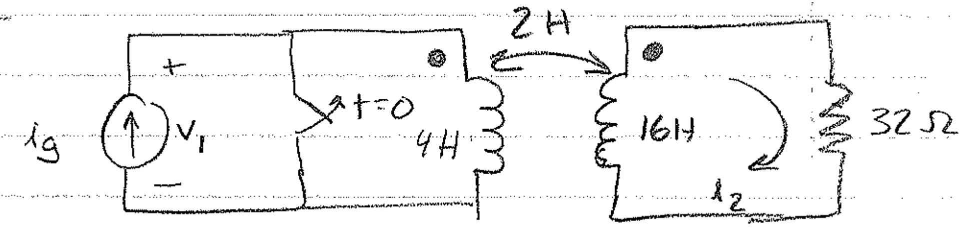
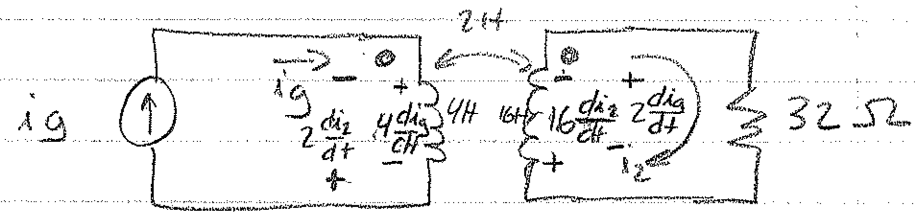
CIRCUITS II

Exam 1 Review

Problem 6-36



(a) Derive the differential Equation that governs the behavior of i_2



KVL (Right Circuit)

$$\boxed{16\frac{di_2}{dt} - 2\frac{di_g}{dt} + 32 i_2 = 0}$$

(b) Show if $i_g = 8 - 8e^{-t}$ A $t \geq 0$, then $i_2 = e^{-t} - e^{-2t}$ A satisfies the equation derived in (a)

$$16\frac{d}{dt}\left(e^{-t} - e^{-2t}\right) - 2\frac{d}{dt}\left(8 - 8e^{-t}\right) + 32\left(e^{-t} - e^{-2t}\right)$$

$$= 16\left(-e^{-t} + 2e^{-2t}\right) - 2\left(8e^{-t}\right) + 32\left(e^{-t} - e^{-2t}\right)$$

$$= -16e^{-t} + 32e^{-2t} - 16e^{-t} + 32e^{-t} - 32e^{-2t} = 0 \checkmark$$

(c) Find an expression for V_1

KVL (Left Circuit)

$$-V_1 + 4\frac{di_g}{dt} - 2\frac{di_2}{dt} = 0$$

$$\boxed{V_1 = 4\frac{di_g}{dt} - 2\frac{di_2}{dt}}$$

2

Problem 6.36

$$v_1 = 4(8e^{-t}) - 2(-e^{-t} + 2e^{-2t})$$

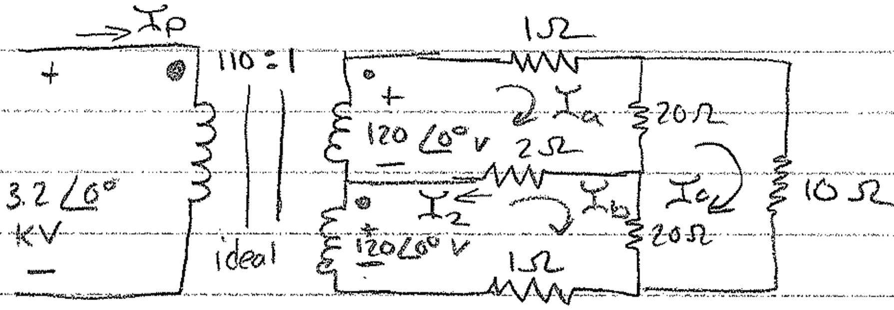
$$\boxed{v_1 = 34e^{-t} - 4e^{-2t} \text{ V}}$$

(d) Find the initial value of v_1

$$\boxed{v_1(0) = 34 - 4 = 30 \text{ V}}$$

③

Problem 9.78



(a) Calculate the branch current in the 2Ω resistor

Mesh Current

Mesh (a)

$$-120\angle 0° + (1)I_a + (20)(I_a - I_c) + 2(I_a - I_b) = 0 \quad (1)$$

Mesh (b)

$$-120\angle 0° + 2(I_b - I_a) + 20(I_b - I_c) + 1(I_b) = 0 \quad (2)$$

Mesh (c)

$$20(I_c - I_a) + 10I_c + 20(I_c - I_b) = 0 \quad (3)$$

$$(1) \Rightarrow \quad 23I_a - 2I_b - 20I_c = 120\angle 0°$$
$$(2) \Rightarrow \quad -2I_a + 23I_b - 20I_c = 120\angle 0°$$
$$(3) \Rightarrow \quad -20I_a - 20I_b + 50I_c = 0$$

Three Equations, 3 Unknowns

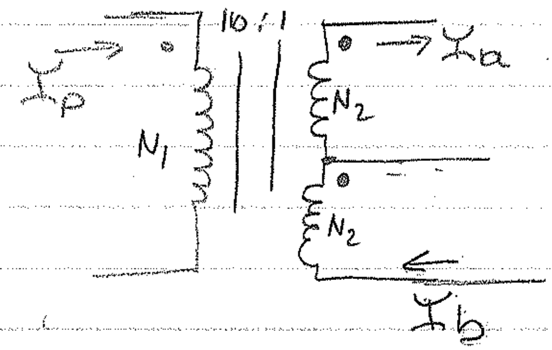
$$I_a = 24\angle 0°\ A, \quad I_b = 24\angle 0°\ A, \quad I_c = 19.2\angle 0°\ A$$

$$I_2 = I_a - I_b = 24\angle 0° A - 24\angle 0° A = \boxed{0}$$

④

Problem 9.78

(b) Calculate I_p, assuming $N_1 I_p = N_2 I_a + N_2 I_b$



$$N_1 I_p = N_2 I_a + N_2 I_b$$

$$I_p = \frac{N_2}{N_1}(I_a + I_b)$$

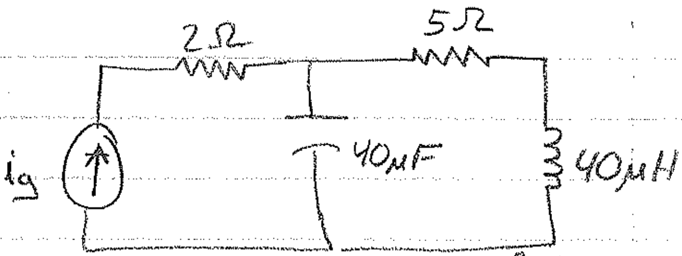
$$I_p = \frac{1}{110}(24\angle 0° + 24\angle 0°)$$

$$\boxed{I_p = 0.436\angle 0°\ A}$$

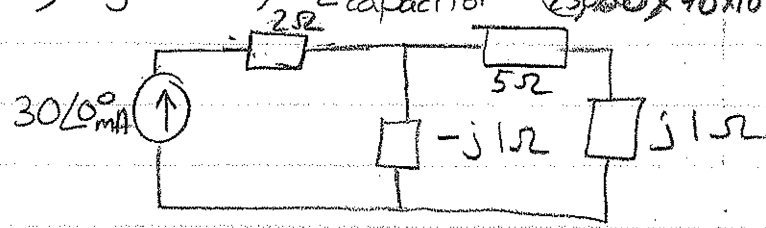
5

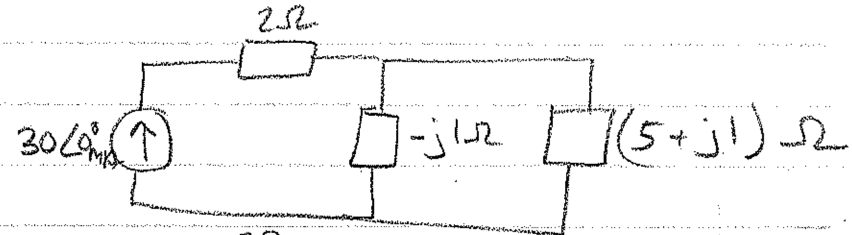
Problem 10.12

$i_g = 30\cos 25{,}000t$ mA

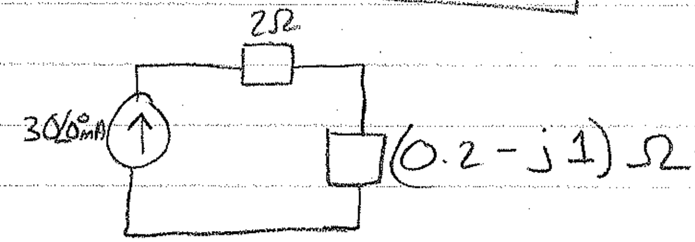


$Z_{inductor} = j(25{,}000)(40\times10^{-6}) = j1\,\Omega$, $Z_{capacitor} = \left(\dfrac{-j}{25{,}000\times40\times10^{-6}}\right) = -j1\,\Omega$





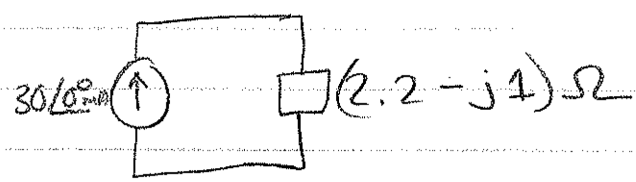
$$Z_{eq} = \left(\frac{1}{-j1} + \frac{1}{5+j1}\right)^{-1}$$



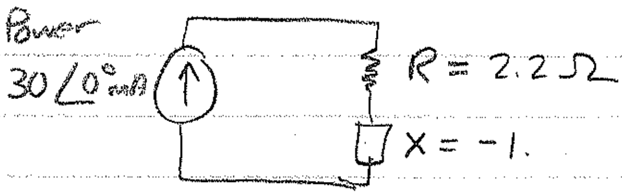
$$= \left(j + \frac{5-j1}{26}\right)^{-1}$$

$$= \left(j + \frac{5}{26} - j\left(\frac{1}{26}\right)\right)^{-1} = (0.1923 + j0.9615)^{-1}$$

$$= \frac{0.1923 - j0.9615}{(0.1923^2 + 0.9615^2)} = (0.2 - j1)\,\Omega$$



Average Power => Real Power



$$P = \frac{1}{2}|I_m|^2 R$$

$$= \frac{1}{2}(30\times10^{-3}\,A)^2(2.2\,\Omega)$$

$$P = 9.9\times10^{-4}\,W = \boxed{990\,\mu W}$$

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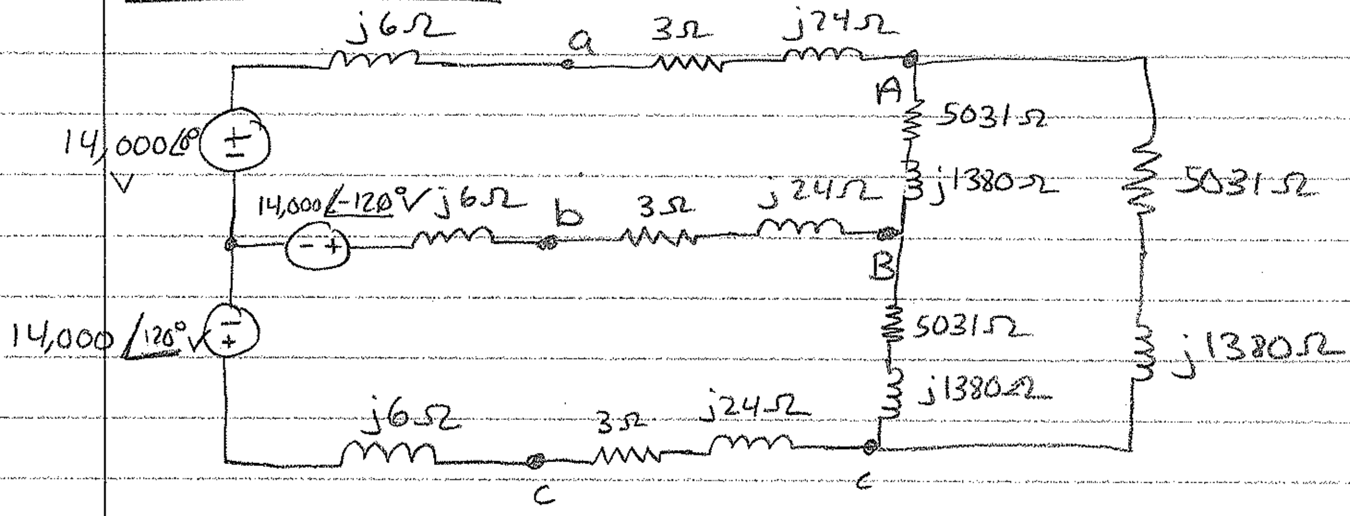
Problem 11, 19



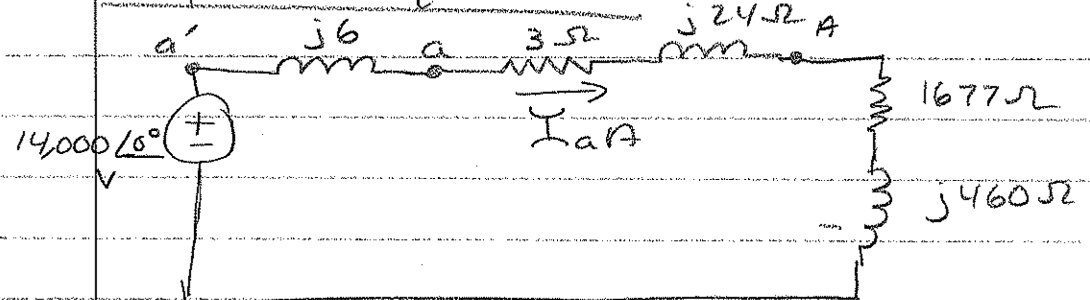
(a) Determine I_{cA} (rms)

Is it balanced? abc sequence, source impedances are equal
Line " " "
Load " " "
=> Yes

WYE - to - DELTA Circuit

$$Z_Y = \frac{Z_\Delta}{3} = \frac{5031\,\Omega + j1380\,\Omega}{3} = (1677 + j460)\,\Omega$$

a-phase equivalent



$$I_{aA} = \frac{14{,}000\angle 0^\circ \text{ V}}{(j6)\,\Omega + (3 + j24)\,\Omega + (1677 + j460)\,\Omega}$$

$$I_{aA} = \frac{14{,}000\angle 0^\circ \text{ V}}{1680 + j490\,\Omega} = \boxed{8\angle -16.26^\circ \text{ A}}$$

$$I_{bB} = 8\angle -16.26 - 120^\circ = \boxed{8\angle -136.26^\circ \text{ A}}$$

$$I_{cC} = 8\angle -16.26 + 120^\circ = \boxed{8\angle 103.74^\circ \text{ A}}$$

(7)

Problem 11.19

$$I_{cC} = \sqrt{3}\,\angle -30^\circ\, I_{CA}$$

$$I_{CA} = \frac{I_{cC}}{\sqrt{3}\angle -30^\circ} = \frac{8\angle 103.74^\circ}{\sqrt{3}\angle -30^\circ} = 4.62\angle 133.74^\circ A$$

$$I_{CA}\,(rms) = \frac{4.62}{\sqrt{2}}\angle 133.74^\circ A = \boxed{3.266\angle 133.74^\circ A}$$

(b) What percentage of the average power delivered by the three-phase source is dissipated in the three phase load

Source

$$S = -\tfrac{1}{2} V\, I_{aA}^{*} = -\tfrac{1}{2}(14{,}000\angle 0^\circ V)(8\angle 16.26^\circ A)$$

$$= -(56000\angle 16.26^\circ)\ VA$$

$$S = -(53{,}760 + j15{,}680)\ VA$$

$$\boxed{P_{del} = 53.76\ kW}$$

Load

$$P_{abs} = \tfrac{1}{2}|I_{aA}|^2 R = \tfrac{1}{2}(8)^2(1677) = 53664\ W$$

$$\boxed{P_{abs} = 53.66\ kW}$$

$$\boxed{\% = \frac{53.66\ kW}{53.76\ kW} \times 100\% = 99.8\%}$$

①

CIRCUITS II

EXAM 2 Review

Find the Laplace Transform of:

$$f(t) = -20e^{-5t}u(t-2)$$

$$= -20e^{-5[(t-2)+2]}u(t-2)$$

$$= -20e^{-5(t-2)}(e^{-10})u(t-2)$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

$$F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-5t}\} = \frac{1}{s+5}$$

$$-20e^{-10}\mathcal{L}\{e^{-5(t-2)}u(t-2)\} = -20e^{-10}e^{-2s}\left(\frac{1}{s+5}\right)$$

$$= \boxed{\frac{(-9.08\times10^{-4})e^{-2s}}{s+5}}$$

$$\mathcal{L}\{\tfrac{d}{dt}\cos\omega t\} \qquad f(0^-) = 0$$

$$= sF(s) - f(0^-)$$

$$= s\left(\frac{s}{s^2+\omega^2}\right) - 0 = \boxed{\frac{s^2}{s^2+\omega^2}}$$

$$\mathcal{L}\{\sin(\omega t+\theta)\}$$

$$\sin(\omega t+\theta) = \sin\omega t\cos\theta + \sin\theta\cos\omega t$$

$$\cos\theta\,\mathcal{L}\{\sin\omega t\} + \sin\theta\,\mathcal{L}\{\cos\omega t\}$$

$$\frac{\omega\cos\theta}{s^2+\omega^2} + \frac{s(\sin\theta)}{s^2+\omega^2} = \boxed{\frac{\omega\cos\theta + s(\sin\theta)}{s^2+\omega^2}}$$

(2)

$$F(s) = \frac{24s^2+10s+5}{s(s^2+5s+6)} = \frac{25s^2+10s+5}{s(s+3)(s+2)}$$

$$\frac{25s^2+10s+5}{s(s+3)(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{s+2}$$

$$K_1 = \left.\frac{25s^2+10s+5}{\cancel{s}(s+3)(s+2)}\right|_{s=0} = \frac{5}{5} = 1$$

$$K_2 = \left.\frac{25s^2+10s+5}{s\cancel{(s+3)}(s+2)}\right|_{s=-3} = \frac{25(9)+10(-3)+5}{(-3)(-3+2)} = 66.67$$

$$K_3 = \left.\frac{25s^2+10s+5}{s(s+3)\cancel{(s+2)}}\right|_{s=-2} = \frac{25(4)+10(-2)+5}{(-2)(-2+3)} = -42.5$$

$$F(s) = \frac{1}{s} + 66.67\left(\frac{1}{s+3}\right) - 42.5\left(\frac{1}{s+2}\right)$$

$$f(t) = \left[1 + 66.67e^{-3t} - 42.5e^{-2t}\right]u(t)$$

$$F(s) = \frac{6s}{s^2+2s+8} = \qquad s = \frac{-2 \pm \sqrt{4-32}}{2} = -1 \pm j5.29$$

$$\frac{6s}{s^2+2s+8} = \frac{K}{(s+1-j5.29)} + \frac{K^*}{(s+1+j5.29)}$$

$$K = \left.\frac{6s}{\cancel{(s+1-j5.29)}(s+1+j5.29)}\right|_{s=-1+j5.29}$$

$$= \frac{6(-1+j5.29)}{(j10.58)} = \frac{-j6-31.74}{-10.58}$$

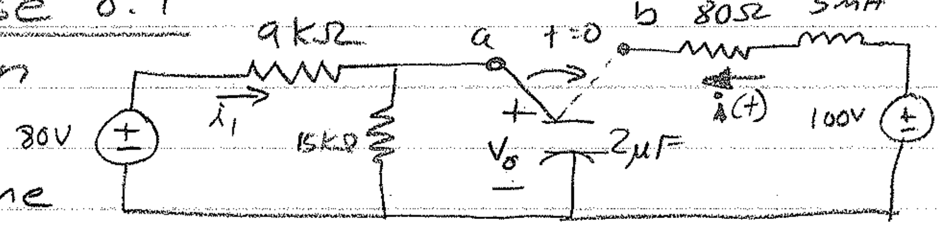
$$K = -3 + j0.567 \Rightarrow 3.05\angle 10.7^\circ$$

$$\boxed{f(t) = 2(3.05)e^{-t}\cos(5.29t + 10.7^\circ)}$$

③

Drill Exercise 8.7

Switch has been in position a for a long time



Find i(t)

No current in the capacitor

$$i_1 = \frac{80V}{9k\Omega + 15k\Omega} = 3.33mA$$

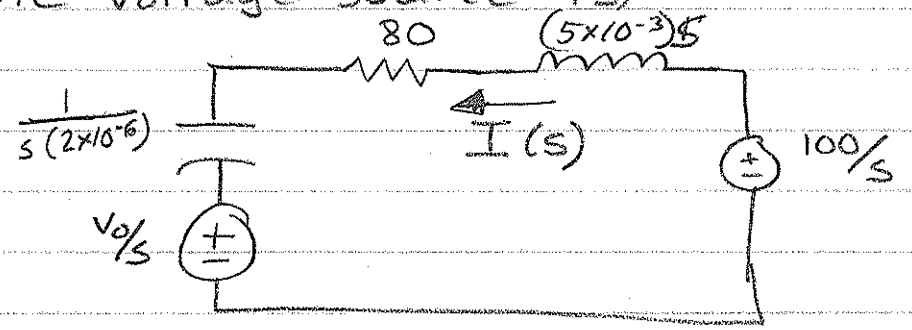
KVL

$$-80 + (9k\Omega)(3.33) + V_o = 0$$

$$\boxed{V_o = 50V}$$

S-Domain for $t \geq 0$

Polarity of V_o tells you what the polarity of the equivalent voltage source is.



KVL

$$\frac{50}{s} + \frac{5\times10^5}{s}I + 80I + (5\times10^{-3})sI - \frac{100}{s} = 0$$

$$50 + 5\times10^5 I + 80sI + (5\times10^{-3})s^2 I - 100 = 0$$

$$I(5\times10^{-3}s^2 + 80s + 5\times10^5) = +50$$

$$I = \frac{450}{(5\times10^{-3})(s^2 + 16000s + 100\times10^6)}$$

$$s_{1,2} = \frac{-16000 \pm \sqrt{(16000)^2 - 4(100\times10^6)}}{2} = -8000 \pm j6000$$

(4)

$$\frac{10000}{(s+8000-j6000)(s+8000+j6000)} = \frac{K}{s+8000-j6000} + \frac{K^*}{s+8000+j6000}$$

$$K = \left.\frac{10,000}{(s+8000-j6000)(s+8000+j6000)}\right|_{s=-8000+j6000}$$

$$K = \frac{10,000}{j12000} = \frac{-j10,000}{12,000} = -j0.833$$

$$K = 0.833\angle{-90^\circ}$$

$$i(t) = \left[2(0.833)e^{-8000t}\cos(6000t-90^\circ)\right]u(t)\ A$$

$$\boxed{i(t) = \left[1.67e^{-8000t}\sin(6000t)\right]u(t)\quad A}$$

1

CIRCUITS II
REVIEW ⇒ EXAM 3

Drill Exercise 14.2

Given

Series RL Low-Pass Filter with cutoff frequency = 2 KHz, R = 5 KΩ

(a) Compute L

Standard Form

$$\omega_c = R/L$$

$$(2\pi)(2000\,Hz) = \frac{5000\,\Omega}{L}$$

$$L = 0.4\,H = \boxed{400\,mH}$$

(b) Find $|H(j\omega)|$ at 50 KHz

$$H(s) = \frac{\omega_c}{s+\omega_c}$$

$$H(j\omega) = \frac{\omega_c}{j\omega + \omega_c}$$

$$|H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2+\omega_c^2}}$$

$$\omega_c = (2\pi)(2000\,Hz) = 12{,}566\ rad/s$$

$$|H(j\omega)| = \frac{12{,}566\ rad/s}{\sqrt{[(2\pi)(50{,}000\,Hz)]^2 + (12{,}566\,rad/s)^2}}$$

$$|H(j\omega)| = 0.04$$

(c) Find $\theta(j\omega)$ @ 50 KHz

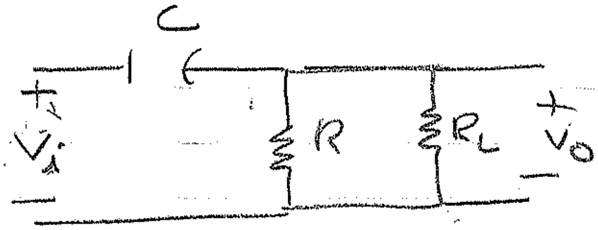
$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$= -\tan^{-1}\left(\frac{(2\pi)(50{,}000)(0.4\,H)}{5000\,\Omega}\right)$$

$$\boxed{\theta(j\omega) = -87.72^\circ}$$

CIRCUITS II
EXAM 3 REVIEW

Drill Exercise 14.5

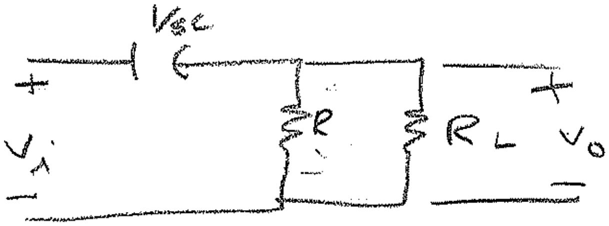


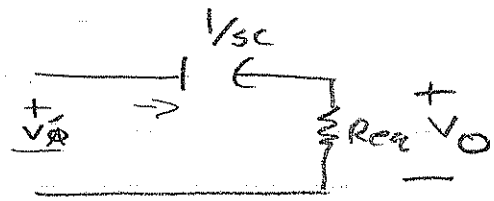
Derive the transfer function for V_o

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R_L}$$

$$\frac{1}{R_{eq}} = \frac{R_L + R}{R R_L}$$

$$R_{eq} = \frac{R R_L}{R + R_L}$$





$$-V_i + \left(\frac{1}{sC} + \frac{R R_L}{R + R_L}\right) I = 0$$

$$I = \frac{V_i}{\left(\frac{1}{sC} + \frac{R R_L}{R + R_L}\right)}$$

$$V_o = \frac{R R_L}{R + R_L}\left(\frac{V_i}{1/sC + \frac{R R_L}{R + R_L}}\right)$$

$$H(s) = \frac{R R_L}{R + R_L}\left(\frac{s}{\frac{1}{C} + s\left(\frac{R R_L}{R + R_L}\right)}\right)$$

$$H(s) = \frac{\left(\frac{R R_L}{R + R_L}\right) s}{\frac{R R_L}{R + R_L}\left(s + \frac{(R + R_L)}{R R_L C}\right)}$$

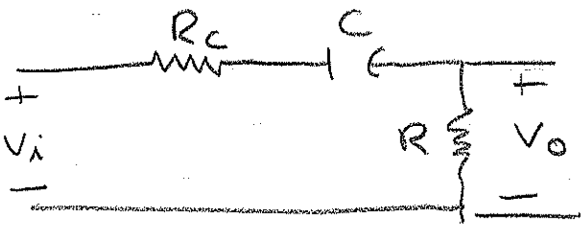
$$\boxed{H(s) = \frac{s}{s + \underbrace{\left(\frac{R + R_L}{R_L}\right)\frac{1}{RC}}_{\omega_c}}}$$

$$\boxed{\omega_c = \frac{R + R_L}{R_L}\left(\frac{1}{RC}\right)}$$

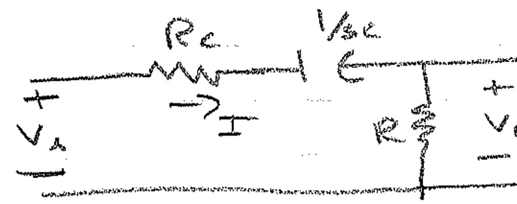
3

CIRCUITS II
EXAM 3 REVIEW

Problem 14.8



(a) Derive the transfer function for V_o



$$-V_i + IR_c + \frac{1}{sC} I + IR = 0$$

$$I = \frac{V_i}{R + R_c + 1/sC} \qquad V_o = IR$$

$$H(s) = \frac{R}{(R+R_c) + \frac{1}{sC}}$$

$$H(s) = \frac{sR}{s(R+R_c) + \frac{1}{C}}$$

$$H(s) = \frac{s\left(R/R+R_c\right)}{s + \frac{1}{(R+R_c)C}}$$

$$\boxed{H(s) = \left(\frac{R}{R+R_c}\right) \frac{s}{s + \frac{1}{(R+R_c)C}}}$$

(b) At what Frequency will the magnitude of $H(j\omega)$ be a maximum

$$\boxed{f = \infty}$$

(c) What is the maximum magnitude of $H(j\omega)$

$$H(j\omega) = \left(\frac{R}{R+R_c}\right) \frac{j\omega}{j\omega + \frac{1}{(R+R_c)C}}$$

$$|H(j\omega)| = \left(\frac{R}{R+R_c}\right) \frac{\omega}{\sqrt{\omega^2 + \left[\frac{1}{(R+R_c)C}\right]^2}} = \left(\frac{R}{R+R_c}\right) \frac{\omega}{\omega \sqrt{1 + \frac{1}{\omega^2}\left[\frac{1}{(R+R_c)C}\right]^2}}$$

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CIRCUITS II
EXAM 3 REVIEW

$$|H(j\omega)| = \frac{R}{R+R_c} \frac{1}{\sqrt{1 + \frac{1}{\omega^2}\left[\frac{1}{(R+R_c)C}\right]^2}}$$

$$\lim_{\omega \to \infty} = \boxed{\frac{R}{R+R_c} = H_{max}}$$

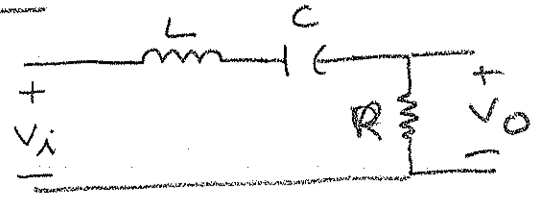
(d) What is ω_c

$$\boxed{\omega_c = \frac{1}{(R+R_c)C}}$$

(5)

CIRCUITS II
EXAM 3 REVIEW

Drill Exercise 14.6



Determine values for R and L such that the center frequency = 12 kHz, and the quality factor = 6. Use 0.1 μF Capacitors.

Standard RLC Bandpass Filter

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$(2\pi)(12{,}000\ \text{Hz}) = \sqrt{\frac{1}{L(0.1\times10^{-6}\ \text{F})}}$$

$$\boxed{L = 1.76\ \text{mH}}$$

$$Q = \sqrt{\frac{L}{CR^2}}$$

$$6 = \sqrt{\frac{1.76\times10^{-3}\ \text{H}}{(0.1\times10^{-6}\ \text{F})(R^2)}}$$

$$\boxed{R = 22.1\ \Omega}$$

CIRCUITS II
EXAM 4 Review

Drill Exercise 15.4

Design a unity-gain Active bandpass filter with a center frequency of 200 Hz, and a bandwidth of 1000 Hz. Use 5 μF capacitors

$$\omega_0 = 2\pi f_0 = 2\pi(200\,\text{Hz}) = 400\pi \text{ rad/s}$$

$$\beta = 2\pi(1000\,\text{Hz}) = 2000\pi$$

Need ω_{c1} & ω_{c2}

$$\beta = \omega_{c2} - \omega_{c1}$$

$$\boxed{\omega_{c2} - \omega_{c1} = 2000\pi} \quad 1$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

$$\omega_{c1}\omega_{c2} = 16\times10^4\pi^2$$

$$\boxed{\omega_{c2} = \frac{16\times10^4\pi^2}{\omega_{c1}}} \quad 2$$

$$\frac{16\times10^4\pi^2}{\omega_{c1}} - \omega_{c1} = 2000\pi$$

$$16\times10^4\pi^2 - \omega_{c1}^2 = (2000\pi)\omega_{c1}$$

$$\omega_{c1}^2 + (2000\pi)\omega_{c1} - 16\times10^4\pi^2 = 0$$

$$\omega_{c1} = \frac{-2000\pi \pm \sqrt{(2000\pi)^2 + (4)(16\times10^4\pi^2)}}{2}$$

$$= -1000\pi \pm 3383.6$$

$$\boxed{\omega_{c1} = 242 \text{ rad/s}} \qquad \boxed{\omega_{c2} = 6525.2 \text{ rad/s}}$$

$$\omega_{c2}/\omega_{c1} = 27$$

2

Drill Exercise 15.4

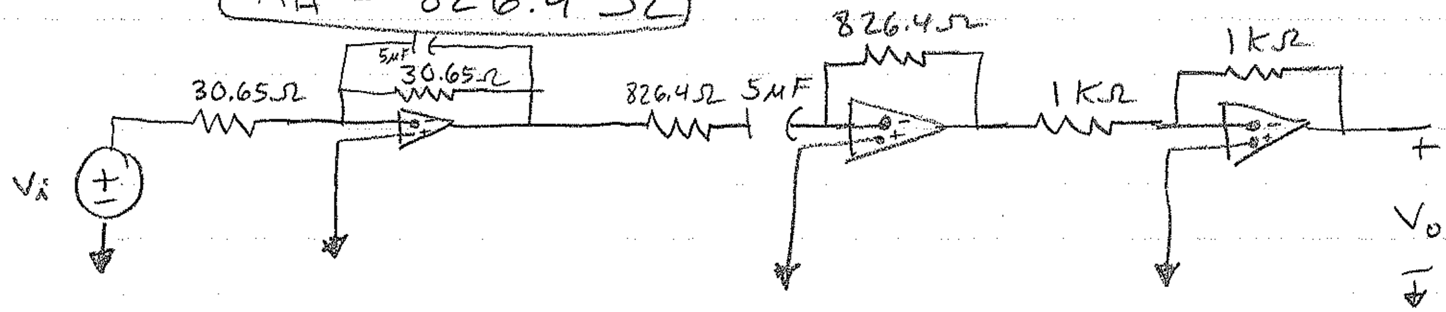
$$\frac{1}{R_L C_L} = \omega_{c2} \Rightarrow 6525.2 = \frac{1}{R_L(5\times10^{-6}F)}$$

$$\boxed{R_L = 30.65\,\Omega}$$

$$\frac{1}{R_H C_H} = \omega_{c1} \Rightarrow 242 = \frac{1}{R_H(5\times10^{-6}F)}$$

$$\boxed{R_H = 826.4\,\Omega}$$

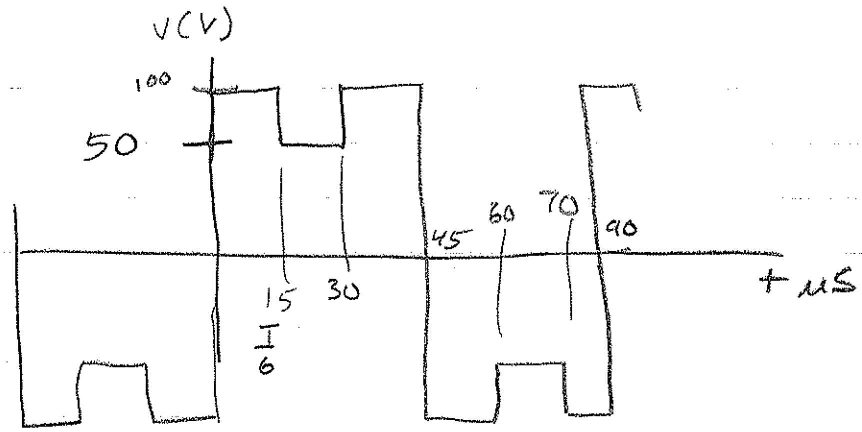
$K = 1 \Rightarrow R_f = R_i = 1k\Omega$



③

Problem 16.8

(a) Find ω_0



$T = 90\,\mu S$

$f = \frac{1}{90\times10^{-6}}$

$= 11{,}111\ Hz$

$\omega_0 = 2\pi(11{,}111\,Hz) = \boxed{69{,}813.2\ rad/s}$

(b) Find f_0 => $f_0 = 11{,}111\ Hz = 11.1\,kHz$

(c) Find a_v

Even or odd
Half Wave? Yes
Quarter Wave? Yes

$a_v = 0$

(d) Find equations for a_k & b_k

$a_k = 0$ for all k, $b_k = 0$ for k even

$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin(k\omega_0 t)\,dt$ k odd $T = 90\,\mu S$ $\frac{15}{90} = \frac{1}{6}$

$\omega_0 = \frac{2\pi}{T}$

$= \frac{8}{T}\int_0^{\frac{T}{6}} 100\sin\left(\frac{2\pi k t}{T}\right)dt + \frac{8}{T}\int_{T/6}^{T/4} 50\sin\left(\frac{2\pi k t}{T}\right)dt$

$= -\left(\frac{800}{T}\right)\left(\frac{T}{2\pi k}\right)\left[\cos\left(\frac{2\pi k t}{T}\right)\right]_0^{T/6} - \left(\frac{400}{T}\right)\left(\frac{T}{2\pi k}\right)\left[\cos\left(\frac{2\pi k t}{T}\right)\right]_{T/6}^{T/4}$

$= -\frac{400}{k\pi}\left(\cos\left(\frac{k\pi}{3}\right) - 1\right) - \frac{200}{k\pi}\left(\underbrace{\cos\left(\frac{k\pi}{2}\right)}_{=0} - \cos\left(\frac{k\pi}{3}\right)\right)$

$b_k = \frac{400}{k\pi} - \frac{200}{k\pi}\cos\frac{k\pi}{3} = \boxed{\frac{200}{k\pi}\left(2 - \cos\left(\frac{k\pi}{3}\right)\right)}$ k odd

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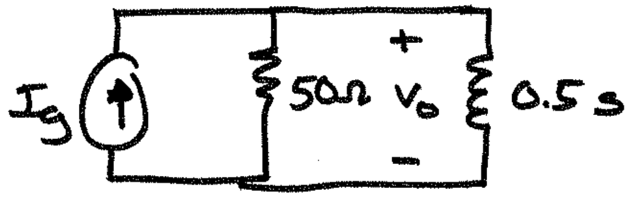
Problem 16.8

(e) Find $v(t)$

$$v(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{200}{n\pi}\left(2-\cos\left(\frac{n\pi}{3}\right)\right)\sin(n\omega_0 t)$$

$$\boxed{v(t) = \frac{200}{\pi}\sum_{n=1,3,5}^{\infty} \frac{1}{n}\left[2-\cos\left(\frac{n\pi}{3}\right)\right]\sin(n\omega_0 t)}$$

17.21)



$$\frac{V_o}{50} + \frac{V_o}{0.5s} = I_g$$

$$V_o = \frac{I_g}{\left(\frac{1}{50} + \frac{2}{s}\right)}$$

$$H(s) = \frac{V_o}{I_g} = \frac{50s}{s+100}$$

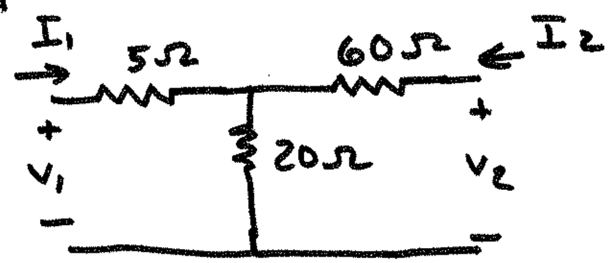
$$H(j\omega) = \frac{j\omega 50}{j\omega + 100} \quad , \quad I_g(\omega) = \frac{4}{j\omega}$$

$$V_o(\omega) = H(j\omega) \cdot I_g(\omega) = \left(\frac{j\omega 50}{j\omega+100}\right)\left(\frac{4}{j\omega}\right)$$

$$= \frac{200}{j\omega + 100}$$

$$\boxed{v_o(t) = 200e^{-100t}u(t) \text{ V}}$$

18.7)



$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$= (5+20)\Omega = \underline{25\Omega}$$

$$z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{20I_1}{I_1} = \underline{20\Omega}$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = 60\Omega + 20\Omega = \underline{80\Omega}$$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{20I_2}{I_2} = \underline{20\Omega}$$

18.9 $\quad a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad V_2 = \frac{V_1}{R_1+R_3} R_3$

$$a_{11} = \frac{R_1+R_3}{R_3} = 1 + \frac{R_1}{R_3} = 1.2 \Rightarrow \frac{R_1}{R_3} = 0.2$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} ; \; V_2 = I_1 R_3$$

$$a_{21} = \frac{1}{R_3} = 20\times10^{-3} \Rightarrow \boxed{R_3 = 50\,\Omega}$$

$$R_1 = 0.2 R_3 = \boxed{10\,\Omega}$$

$$a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0} \qquad I_2 = \frac{-I_1 R_3}{R_3+R_2}$$

$$a_{22} = \frac{R_2+R_3}{R_3} = 1.4 \Rightarrow \frac{R_2}{R_3} = 0.4 \Rightarrow R_2 = 0.4R_3 = \boxed{20\,\Omega}$$

$$\boxed{R_1 = 10\,\Omega,\; R_2 = 20\,\Omega,\; R_3 = 50\,\Omega}$$

18.12) Port 1 open ($I_1=0$) $\Rightarrow$ $V_1 = 1\text{mV}$, $V_2 = 10\text{V}$, $I_2 = 200\mu A$

Port 1 short-circuited ($V_1=0$) $\Rightarrow$ $I_1 = -0.5\mu A$, $I_2 = 80\mu A$, $V_2 = 5V$

a) $V_1 = a_{11}V_2 - a_{12}I_2$

$I_1 = a_{21}V_2 - a_{22}I_2$

$I_1 = 0 \Rightarrow \quad 1\times10^{-3} = a_{11}(10) - a_{12}(200\times10^{-6}) \qquad (1)$

$V_1 = 0 \Rightarrow \quad 0 = a_{11}(5) - a_{12}(80\times10^{-6}) \qquad (2)$

Solving $\Rightarrow$ $\cancel{a_{21} = -5\times10^{-7}\,\Omega^{-1}} \quad \cancel{a_{22} = -0.025}$

$$\boxed{a_{11} = -4\times10^{-4} \qquad a_{12} = -25\,\Omega}$$

$I_1 = 0 \Rightarrow \quad 0 = a_{21}(10) - a_{22}(200\times10^{-6}) \qquad (3)$

$V_1 = 0 \quad -0.5\times10^{-6} = a_{21}(5) - a_{22}(80\times10^{-6}) \qquad (4)$

Solving $\Rightarrow$ $\boxed{a_{21} = -5\times10^{-7}\,\Omega^{-1} \qquad a_{22} = -0.025}$

18.72(b) $b_{11} = \frac{V_2}{V_1}\Big|_{I_1=0} = \frac{1.0}{1\times10^{-3}} = 10{,}000$

$$b_{12} = -\frac{V_2}{I_1}\Big|_{V_1=0} = \frac{-5}{-0.5\times10^{-6}} = 1\times10^{7}\,\Omega$$

$$b_{21} = \frac{I_2}{V_1}\Big|_{I_1=0} = \frac{200\times10^{-6}}{1\times10^{-3}} = 0.2\,\Omega^{-1}$$

$$b_{22} = -\frac{I_2}{V_1}\Big|_{I_1=0} = \frac{-80\times10^{-6}}{-0.5\times10^{-6}} = 160$$

$$\Delta b = \begin{vmatrix} 10{,}000 & 1\times10^{7} \\ 0.2 & 160 \end{vmatrix} = -4\times10^{5}$$

$$h_{11} = \frac{b_{12}}{b_{11}} = \frac{1\times10^{7}}{10{,}000} = 1000\,\Omega,\quad h_{12} = \frac{1}{b_{11}} = 1\times10^{-4}$$

$$h_{21} = -\frac{\Delta b}{b_{11}} = \frac{-(-4\times10^{5})}{10{,}000} = 40\,\Omega,\quad h_{22} = \frac{b_{21}}{b_{11}} = \frac{0.2}{10{,}000} = 2\times10^{-5}\,\Omega^{-1}$$

$$\Delta h = \begin{vmatrix} 1000 & 1\times10^{-4} \\ 40 & 2\times10^{-5} \end{vmatrix} = 0.016$$

$$a_{11} = -\frac{\Delta h}{h_{21}} = -\frac{0.016}{40} = \boxed{-4\times10^{-4}}$$

$$a_{12} = -\frac{h_{11}}{h_{21}} = \frac{-1000}{40} = \boxed{-25\,\Omega}$$

$$a_{21} = -\frac{h_{22}}{h_{21}} = \frac{-2\times10^{-5}}{40} = \boxed{-5\times10^{-7}\,\Omega^{-1}}$$

$$a_{22} = -\frac{1}{h_{21}} = -\frac{1}{40} = \boxed{-0.025}$$