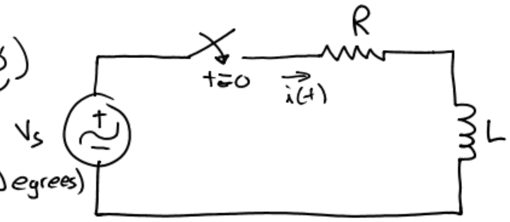
$$v_s = V_m \cos(\underbrace{\omega t + \phi}_{\text{radians}})$$



ϕ = Phase Angle (Degrees)

ω = Angular Frequency (rad/s)

KVL

$$-v_s + iR + L\frac{di}{dt} = 0$$

$$i(t) = \underbrace{\frac{-V_m}{\sqrt{R^2+\omega^2 L^2}} \cos(\phi - \theta) e^{-\left(\frac{R}{L}\right)t}}_{\text{Transient Component}} + \underbrace{\frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \left(\cos(\omega t + \phi - \theta)\right)}_{\text{Steady State}}$$

θ = Angle whose tangent is $\frac{\omega L}{R}$

Transient Component
- Goes to zero after a “long” time

Steady State Solution
- Exists as long as the switch is closed
- Sinusoidal function
- Amplitude is different between the source and response
- The phase angle also is different
- The angular frequency is the same

Phasors
- Complex numbers that contain information about the amplitude and phase angle of a sinusoidal function
- Can use phasors as long as the angular frequencies are the same between the source and the response

Phasor Transform

$$\mathcal{P}\{V_m \cos(\omega t + \phi)\} = V_m e^{j\phi}$$

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

$$\boxed{\mathbf{V} = V_m e^{j\phi} = V_m \cos\phi + j V_m \sin\phi}$$

phasor

$$e^{j\phi} \equiv \angle \phi^\circ$$

$$v = 100 \cos(\omega t - 50^\circ)$$

$$\mathbf{V} = 100 \angle -50^\circ$$

Inverse Phasor Transform

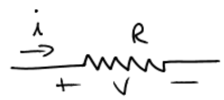
$$\mathcal{P}^{-1}\{20\angle 40^\circ\} = 20\cos(\omega t + 40^\circ)$$

rms Value

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

The Phasor Transform for a Resistor

If $i(t) = I_m\cos(\omega t + \theta)$



$$v(t) = I_m R\cos(\omega t + \theta)$$

$$\mathbf{I} = I_m\angle\theta^\circ$$

phasor current

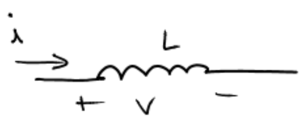
$$\mathbf{V} = I_m R\angle\theta^\circ$$

$$\mathbf{V} = R\underbrace{I_m\angle\theta^\circ}_{\mathbf{I}}$$

$$\boxed{\mathbf{V} = R\,\mathbf{I}}$$

The Phasor Transform for an Inductor

If $i(t) = I_m\cos(\omega t + \theta)$



$$\begin{aligned} v(t) &= L\frac{di}{dt} \\ &= L I_m(-\omega\sin(\omega t + \theta)) \\ &= -\omega L I_m \sin(\omega t + \theta) \\ &= -\omega L I_m[-\cos(\omega t + \theta + 90^\circ)] \\ &= \omega L I_m\cos(\omega t + \theta + 90^\circ) \end{aligned}$$

$$\left(\begin{array}{l}\mathbf{V} = \omega L I_m \angle \theta + 90^\circ \\ \mathbf{I} = I_m \angle \theta^\circ\end{array}\right)$$

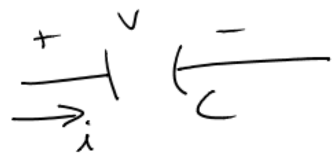
$$\mathbf{V} = j\omega L \underbrace{I_m \angle \theta}_{\mathbf{I}}$$

$$\boxed{\mathbf{V} = j\omega L\, \mathbf{I}}$$

$$\begin{aligned} e^{j(\theta+90^\circ)} &= e^{j\theta} e^{j90} \\ &= e^{j\theta}(\cos 90^\circ + j\sin 90^\circ) \\ &= j e^{j\theta} = j \angle \theta \end{aligned}$$

The Phasor Transform for a Capacitor

If $v(t) = V_m \cos(\omega t + \theta)$



$$\begin{aligned} i(t) &= C \frac{dv}{dt} \\ &= -C\, \omega V_m \sin(\omega t + \theta) \\ &= -\omega C V_m \left(-\cos(\omega t + \theta + 90^\circ)\right) \end{aligned}$$

$$i(t) = \omega C V_m \cos(\omega t + \theta + 90^\circ)$$

$$\mathbf{I} = \omega C V_m \angle \theta + 90^\circ$$

$$\mathbf{V} = V_m \angle \theta^\circ$$

$$\mathbf{I} = j\omega C \underbrace{V_m \angle \theta^\circ}_{\mathbf{V}}$$

$$\mathbf{I} = j\omega C\, \mathbf{V}$$

$$\boxed{\mathbf{V} = \frac{1}{j\omega C} \mathbf{I}}$$

Impedance

$$\mathbf{V} = \underbrace{Z}_{\text{Impedance}}\mathbf{I}$$

Resistor

$$Z_R = R$$

Inductor

$$Z_L = j\omega L$$

Capacitor

$$Z_C = \frac{1}{j\omega C}\left(\frac{j}{j}\right) \Rightarrow Z_C = \frac{-j}{\omega C}$$

$\mathbf{V} = Z\mathbf{I} \Rightarrow$ All resistive circuit analysis methods apply to impedances