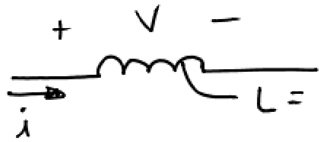
ENGR 2219 – Linear Circuits I
Inductors and Capacitors

Inductors
- Oppose any change in current
- A basic inductor consists of a coil of wire wound around a magnetic or non-magnetic core
- Behavior is associated with magnetic fields



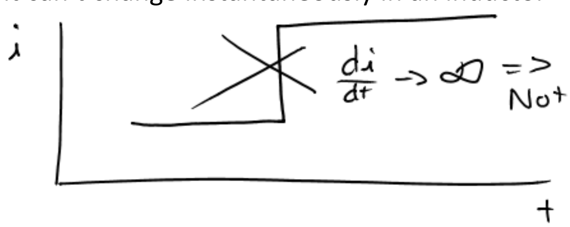
L = Inductance (Henry, H)

- Characteristic Equation

$$\boxed{v = L\frac{di}{dt}}$$

$$i \Rightarrow \text{constant} \Rightarrow \frac{di}{dt} = 0 \Rightarrow v = 0$$

- Current can't change instantaneously in an inductor



$\frac{di}{dt} \to \infty \Rightarrow v \to \infty$
Not possible

- Current in terms of voltage

$$v = L\frac{di}{dt}$$

Multiply both sides by dt

$$v\,dt = L\,di$$

Integrate both sides

$$\int_{t_0}^{t} v\,dt = \int_{i(t_0)}^{i(t)} L\,di$$

$$= L\left(i(t) - i(t_0)\right)$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)}$$

- Power

$$p = vi$$

$$p = \left(L\frac{di}{dt}\right)i$$

$$\boxed{p = Li\frac{di}{dt}}$$

or

$$\boxed{p = v\left[\frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)\right]}$$

- Energy

Energy

$$p = \frac{dw}{dt}$$

$$Li\frac{di}{dt}$$

$$Li\frac{di}{\cancel{dt}} = \frac{dw}{\cancel{dt}}$$

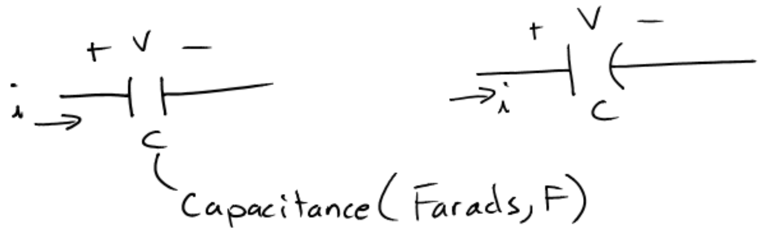
$$\int_0^i Li\,di = \int_0^w dw$$

$$\frac{1}{2}Li^2 = w$$

$$\boxed{w = \frac{1}{2}Li^2}$$

Capacitors

- Two conductors separated by an insulator
- Behavior is associated with electric fields
- The electric field produces a displacement current
    - At the terminals of the capacitor, the displacement current is indistinguishable from the conduction current



- Characteristic Equation

$$\boxed{i = C\frac{dv}{dt}}$$

$$v = \text{constant} \Rightarrow \frac{dv}{dt} = 0 \Rightarrow i = 0$$

- Voltage can't change instantaneously in a capacitor

- Voltage in terms of current

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

- Power

$$p = vi$$

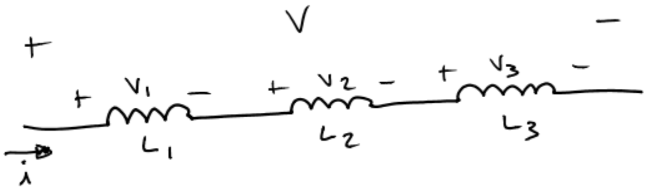
$$\boxed{p = Cv\frac{dv}{dt} \quad \text{or} \quad p = i\left[\frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)\right]}$$

- Energy

$$p = \frac{dw}{dt} \qquad \boxed{w = \frac{1}{2}Cv^2}$$

Series and Parallel Combinations of Inductors and Capacitors

- Inductors in Series



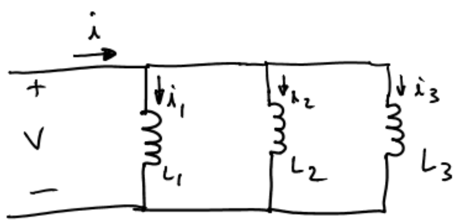
$$V = v_1 + v_2 + v_3$$

$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$V = \underbrace{(L_1 + L_2 + L_3)}_{L_{eq}} \frac{di}{dt}$$

$$\boxed{L_{eq} = \Sigma L_n}$$

- Inductors in Parallel



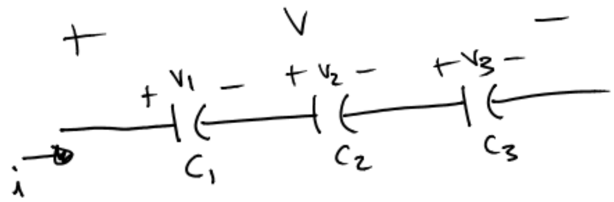
$$i = i_1 + i_2 + i_3$$

$$i = \left[\frac{1}{L_1}\int v\,dt + i_1(t_0)\right] + \left[\frac{1}{L_2}\int v\,dt + i_2(t_0)\right]$$
$$+ \left[\frac{1}{L_3}\int v\,dt + i_3(t_0)\right]$$

$$i = \underbrace{\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)}_{\frac{1}{L_{eq}}} \int v\,dt + \underbrace{\left[i_1(t_0) + i_2(t_0) + i_3(t_0)\right]}_{i(t_0)}$$

$$\boxed{\frac{1}{L_{eq}} = \Sigma \frac{1}{L_n} \Rightarrow L_{eq} = \left(\Sigma \frac{1}{L_n}\right)^{-1}}$$

- Capacitors in Series



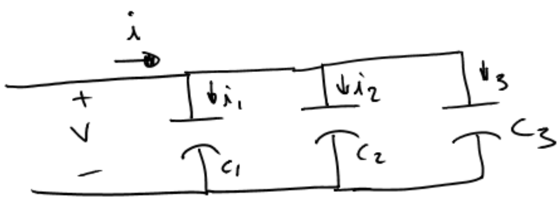
$$V = v_1 + v_2 + v_3$$

$$= \left[\frac{1}{C_1}\int i\,dt + v_1(t_0)\right] + \left[\frac{1}{C_2}\int i\,dt + v_2(t_0)\right] + \left[\frac{1}{C_3}\int i\,dt + v_3(t_0)\right]$$

$$V = \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}_{\frac{1}{C_{eq}}}\int i\,dt + \underbrace{\left[v_1(t_0) + v_2(t_0) + v_3(t_0)\right]}_{v(t_0)}$$

$$\boxed{\frac{1}{C_{eq}} = \Sigma \frac{1}{C_n} \qquad C_{eq} = \left(\Sigma \frac{1}{C_n}\right)^{-1}}$$

- Capacitors in Parallel



$$\boxed{C_{eq} = \Sigma C_n}$$