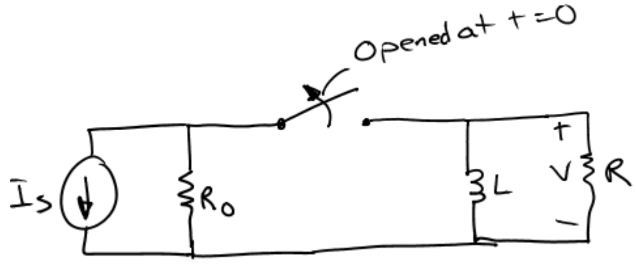
ENGR 2219 – Linear Circuits I

RL and RC Circuits

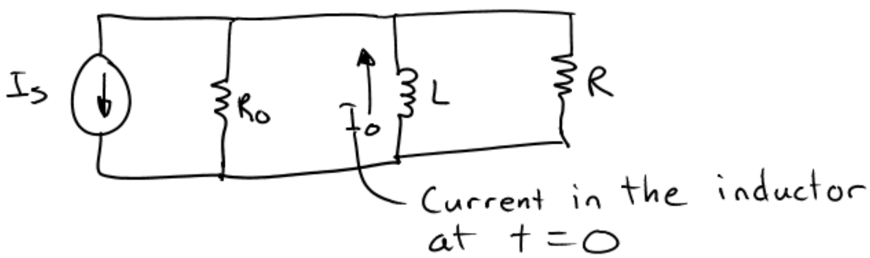
Natural Response

- Behavior is free of excitation (sources)

<u>Natural Response of an RL Circuit</u>



- Prior to t = 0
    - The switch has been closed for a "long" time such that all currents and voltages have reached constant values



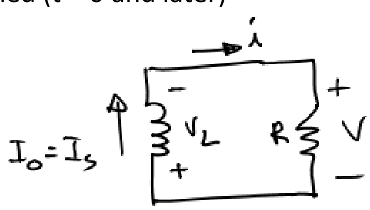
Inductor => "long" time => $\frac{di}{dt} = 0$ => $v_{inductor} = 0$

=> R_0 + R are in parallel with L

$v_{R_0} + v_R = 0$ before the switched is opened

$I_0 = I_s$

- Then the switch is opened (t = 0 and later)



<u>KVL</u>

$$v_L + v = 0$$

$$L\frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} = -\frac{R}{L} i$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}\, dt$$

$$\ln(i(t)) - \ln(I_0) = -\frac{R}{L} t$$

$$\ln\left(\frac{i(t)}{I_0}\right) = -\frac{R}{L} t$$

$$\frac{i(t)}{I_0} = e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$$

- Time constant

$$\tau = \frac{L}{R} \qquad i(t) = I_0 e^{-t/\tau}$$

- Voltage in the resistor

$$V_R = iR$$

$$V_R = I_0 R e^{-t/\tau}$$

- Voltage in the inductor

$$V_L = L\frac{di}{dt} \quad \text{or} \quad V_L + V_R = 0$$

$$\boxed{V_L = -I_0 R e^{-t/\tau}}$$

- Power in the resistor

$$P = i^2 R = \frac{V_R^2}{R} = V_R i$$

$$\boxed{P = I_0^2 R e^{-2t/\tau}}$$

- Power in the inductor

$$P_{inductor} = L i \frac{di}{dt} = v_L i$$

$$\boxed{P_{inductor} = -I_0^2 R e^{-2t/\tau}}$$

- Energy in the resistor

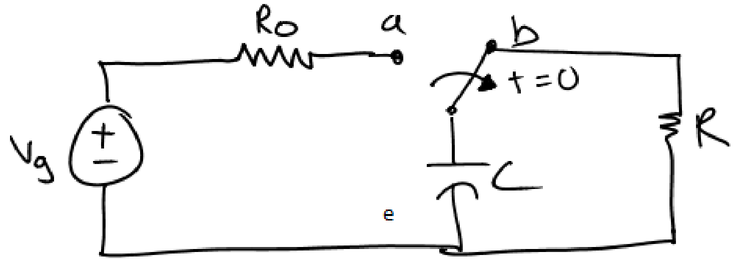
$$p = \frac{dw}{dt} \qquad dw = p\,dt$$

L/R

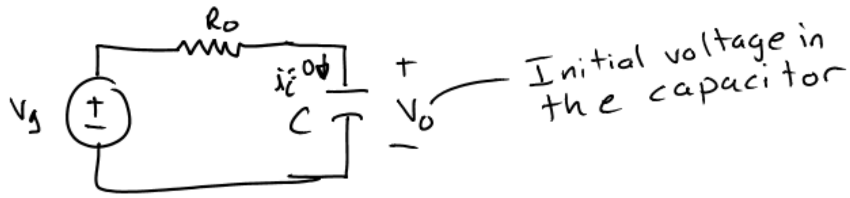
$$w = \int_0^t I_0^2 R e^{-2t/\tau} dt$$

$$w = \frac{1}{2} L I_0^2 \left(1 - e^{-2t/\tau}\right)$$

Natural Response of an RC Circuit

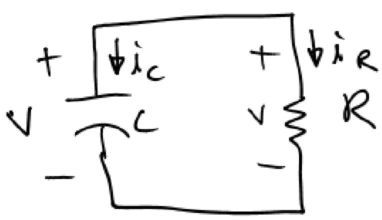


- Prior to t = 0
    - The switch has been at position ‘a’ for a “long” time
    - The current in the capacitor is then zero



$$V_0 = V_g$$

- Then the switch is moved to position ‘b’



$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\int_{V_0}^{v} \frac{dv}{v} = \int_0^t -\frac{1}{RC}\,dt$$

$$\boxed{v(t) = V_0 e^{-\frac{1}{RC}t}}$$

$$\tau = RC \qquad v(t) = V_0 e^{-t/\tau}$$

- Current in the resistor

$$i_R = \frac{v}{R} = \frac{V_0}{R} e^{-t/\tau}$$

- Current in the capacitor

$$i_C = -i_R = C\frac{dv}{dt}$$

$$\boxed{i_C = -\frac{V_0}{R} e^{-t/\tau}}$$

- Power in the resistor

$$P_R = i^2 R = \frac{v^2}{R} = vi$$

$$\boxed{P_R = \frac{V_0^2}{R} e^{-2t/\tau}}$$

- Power in the capacitor

$$P_C = -P_R = -\frac{V_0^2}{R} e^{-2t/\tau}$$

- Energy in the resistor

$$w_R = \int_0^t p\,dt$$

$$\boxed{w_R(t) = \frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right)}$$

- Energy in the capacitor

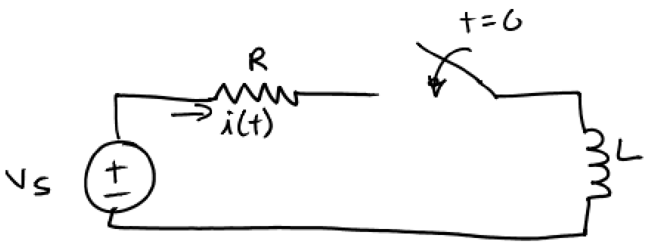
$$w_C(t) = \frac{1}{2} C V^2$$

$$\boxed{w_C(t) = \frac{1}{2} C V_0^2 e^{-2t/\tau}}$$

Step Response

- A DC voltage or current source is connected

Step Response of an RL Circuit

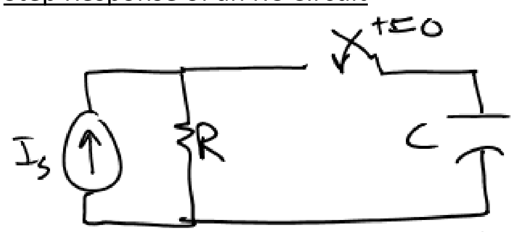


Initial Current in the inductor = I_0

Long time (opened) => $I_0 = 0$

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}, \quad \tau = \frac{L}{R}}$$

Step Response of an RC Circuit



Initial Voltage in the capacitor = V_0

$$\boxed{v(t) = I_s R + \left(V_0 - I_s R\right)e^{-t/\tau}, \quad \tau = RC}$$

General Solution for Both Step and Natural Responses of RL and RC Circuits

$$x(t) = x_f + \left(x(t_0) - x_f\right)e^{-(t-t_0)/\tau}$$

x_f => Final value of x
$x(t_0)$ => Initial value of x
t_0 => Initial time