Control Volume: Region of space though which mass flows in and out

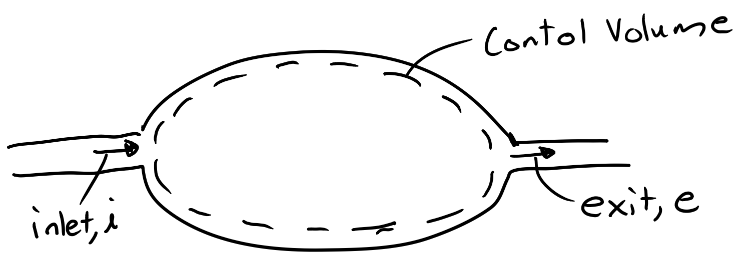
Conservation of Mass for a Control Volume

time rate

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

inlet exit

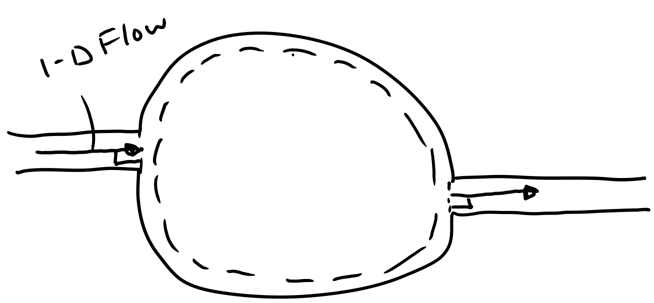
time rate of change of mass contained within the control volume



For multiple inlets and exits

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$$

One-Dimensional Flow
- Flow is normal to the boundary at all locations where mass enters and exits the control volume
- All properties are uniform with position over each inlet and exit area (no change along the cross-section)



$$\dot{m} = \rho A v \quad \text{(1-D Flow)} \qquad \rho = \frac{1}{v}$$

Density

$$\boxed{\dot{m} = \frac{(Av)}{v}}$$ Volumetric Flow Rate

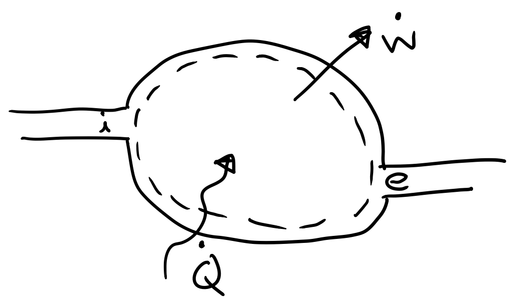
$$\frac{dm_{cv}}{dt} = \sum\left(\frac{A_i v_i}{v_i}\right) - \sum\left(\frac{A_e v_e}{v_e}\right)$$

Steady-State Flow

$$\frac{dm_{cv}}{dt} = 0$$

$$\boxed{\sum \dot{m}_i = \sum \dot{m}_e \quad \text{(steady-state)}}$$

Conservation of Energy for a Control Volume



Energy at the Inlet

$$U_i + \frac{1}{2}m_i v_i^2 + m_i g z_i$$
$$= m u_i + \frac{1}{2}m_i v_i^2 + m_i g z_i$$
$$= m_i\left(u_i + \frac{1}{2}v_i^2 + g z_i\right)$$

Rate of Energy Change

$$\dot{m}_i\left(u_i + \frac{1}{2}v_i^2 + g z_i\right)$$

For the Exit

$$\dot{m}_e\left(u_e + \frac{1}{2}v_e^2 + g z_e\right)$$

Energy Rate Balance

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(u_i + \frac{1}{2}v_i^2 + g z_i\right) - \dot{m}_e\left(u_e + \frac{1}{2}v_e^2 + g z_e\right)$$

Work Components for the Control Volume
1.) Fluid pressure at the inlets and exits
2.) Everything else

Fluid Pressure

$$W_1 = \text{Force} \times \text{Distance}$$
$$= (pA) \times s$$

$$\dot{W}_1 = pA \underbrace{\left(\frac{ds}{dt}\right)}_{\mathrm{v}}$$

$$\dot{W}_1 = p\underbrace{A\mathrm{v}} \qquad \dot{m} = \frac{A\mathrm{v}}{v} \qquad A\mathrm{v} = \dot{m}v$$

$$\dot{W}_1 = p\dot{m}v$$

Everything Else

$$\dot{W}_2 = \dot{W}_{cv}$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \left(\dot{W}_{cv} - p_i\dot{m}_i v_i + p\dot{m}_e v_e\right) + \dot{m}_i\left(u_i + \tfrac{1}{2}\mathrm{v}_i^2 + gz_i\right)$$
$$- \dot{m}_e\left(u_e + \tfrac{1}{2}\mathrm{v}_e^2 + gz_e\right)$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\big(\underbrace{u_i + pv_i}_{h_i} + \tfrac{1}{2}\mathrm{v}_i^2 + gz_i\big) - \dot{m}_e\big(\underbrace{u_e + pv_e}_{h_e} + \tfrac{1}{2}\mathrm{v}_e^2 + gz_e\big)$$

$$\boxed{\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_i + \tfrac{1}{2}\mathrm{v}_i^2 + gz_i\right) - \dot{m}_e\left(h_e + \tfrac{1}{2}\mathrm{v}_e^2 + gz_e\right)}$$

Steady State

$$\frac{dE_{cv}}{dt} = 0$$

$$\boxed{0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(h_i + \tfrac{1}{2}\mathrm{v}_i^2 + gz_i\right) - \dot{m}_e\left(h_e + \tfrac{1}{2}\mathrm{v}_e^2 + gz_e\right)}$$

One Inlet, One Exit

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

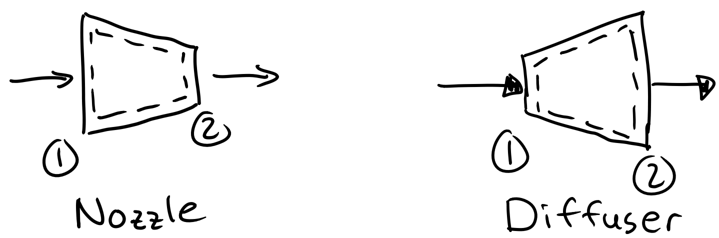
① => Inlet
② => Exit

$$\boxed{0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \tfrac{1}{2}\left(\mathrm{v}_1^2 - \mathrm{v}_2^2\right) + g(z_1 - z_2)\right]}$$

Control Volume Components

Nozzles and Diffusers

- Varying cross-sectional area
- Nozzle: Area decreases in the direction of the flow (pressure and velocity increases)
- Diffuser: Area increases in the direction of the flow (pressure and velocity decreases)

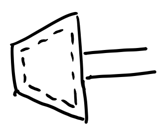


Typically

$$\dot{W}_{cv} = \Delta PE = 0$$

Turbines

- Gas or liquid passing through a set of blades connected to a rotating shaft

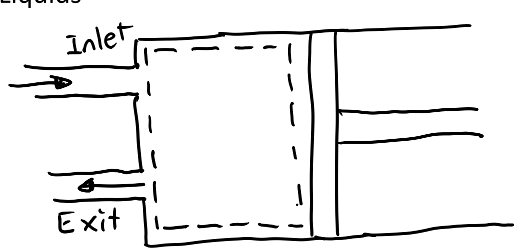


Typically

$\dot{Q}_{cv}$, ΔKE, ΔPE (except hydroelectric) are small with respect to Work and Enthalpy

Compressors and Pumps

- Compressor: Work is done on a gas passing through the compressor to raise the pressure
- Pumps: Liquids



Typically

Same as turbines but there could be heat transfer

Heat Exchangers
- Energy is transferred by heat between fluids at different temperatures

Typically
- If the heat exchanger is modeled as a whole (both fluids), then $\dot{Q}_{cv}$ is small. If not, $\dot{Q}_{cv}$ is significant
- $\dot{W}_{cv}$, ΔKE, ΔPE are small

Throttling Devices
- Restriction in the line to reduce pressure (partially opened valve or a porous plug)

Typically
- $\dot{Q}_{cv}$, $\dot{W}_{cv}$, ΔPE are small
- ΔKE => High velocity at the restriction, but much smaller further away

System Integration (Vapor Power Plant)