5.1) Entropy

Clausius Inequality

$$\oint \left(\frac{\delta Q}{T}\right)_b \leq 0$$

b — Boundary

$\oint$ — Integral across all parts of the boundary

δQ = Heat Transfer along the boundary

T = Temperature at the boundary

$$\oint \left(\frac{\delta Q}{T}\right)_b = -\sigma_{cycle}$$

σ_{cycle} — strength of the inequality

$\sigma_{cycle} = 0$ => No irreversibilities present

$\sigma_{cycle} > 0$ => Irreversibilities present

$\sigma_{cycle} < 0$ => Not possible

Entropy for an Internally Reversible Process

$$\sigma_{cycle} = 0$$

$$dS = \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}}$$

S = Entropy (J/K , Btu/°R)

s = specific entropy (J/kg·K , Btu/lb·°R)

$$s = s_f + x(s_g - s_f)$$

$$\boxed{\begin{aligned} S_2 - S_1 &= \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} \\ Q_{\substack{int\\rev}} &= \int_1^2 T\,dS \end{aligned}}$$

If isothermal

$$\boxed{Q_{\substack{int\\rev}} = T(S_2 - S_1)}$$

T-dS Equations

- Relate entropy to internal energy and specific heats

Air as an Ideal Gas

$$\boxed{s_2 - s_1 = s_2^{o} - s_1^{o} - R\ln\left(\frac{P_2}{P_1}\right)}$$

$s^{o} \Rightarrow$ Table A-22

$s^{o} \neq s$

Entropy Balance for a Closed System

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

Entropy Production

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}}$$

Entropy Rate Balance for a Control Volume

$$\left(\frac{dS}{dt}\right)_{cv} = \sum\left(\frac{\dot{Q}}{T}\right) + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Entropy Rate Production

Steady State

$$0 = \sum \frac{\dot{Q}}{T} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{\sigma}_{cv}$$

One Inlet, One Exit

$$0 = \sum \frac{\dot{Q}}{T} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$