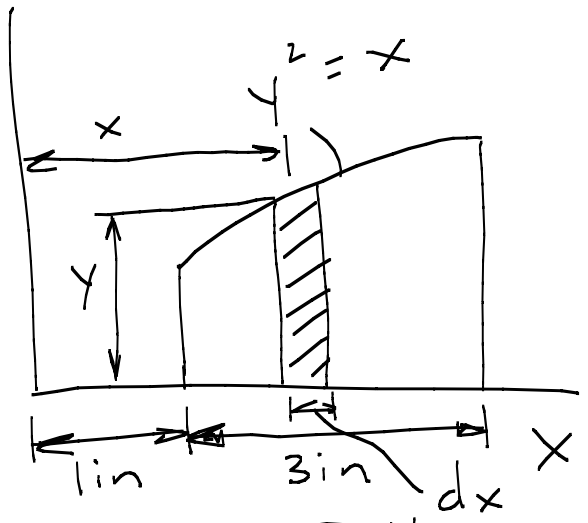
Problem 9.27

Determine $\bar{x}$



$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \tilde{x} = x \qquad dA = y\,dx$$

$$y = x^{1/2}$$

$$\int_A \tilde{x}\, dA = \int_1^4 xy\,dx$$

$$= \int_1^4 x\left(x^{1/2}\right) dx \qquad \int_1^4 x^{3/2}\,dx = \left[\frac{2}{5}x^{5/2}\right]_1^4$$

$$= 12.4 \text{ in}^3$$

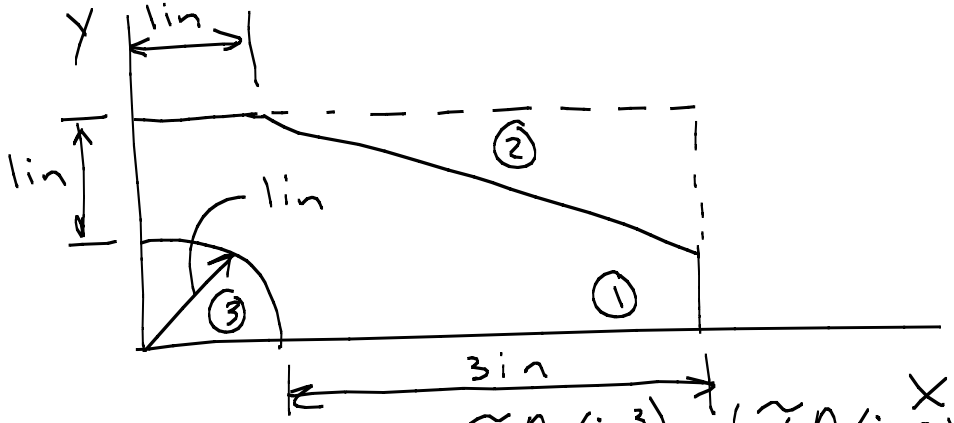
$$\int_A dA = \int_1^4 y\,dx = \int_1^4 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big]_1^4$$

$$= 4.67 \text{ in}^2$$

$$\bar{x} = \frac{12.4}{4.67} = \boxed{2.66 \text{ in}}$$

Problem 9.55

Determine the centroid $(\bar{x}, \bar{y})$



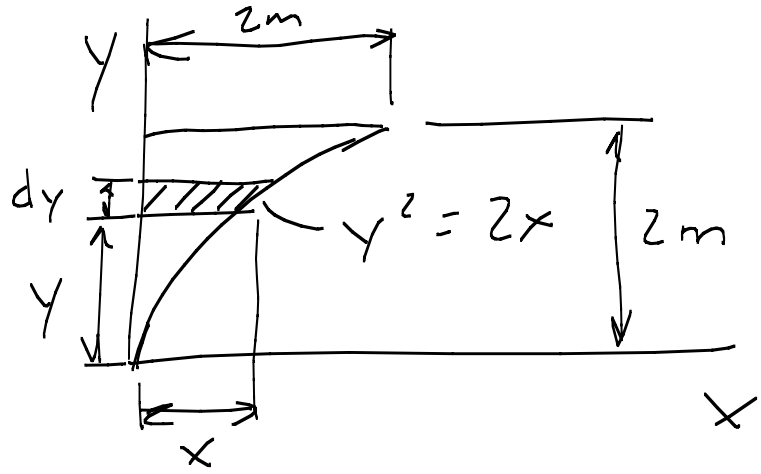
Shape	$A(in^2)$	$\tilde{x}(in)$	$\tilde{y}(in)$	$\tilde{x}A(in^3)$	$\tilde{y}A(in^3)$
①	$(4)(2) = 8$	$\frac{1}{2}(4) = 2$	$\frac{1}{2}(2) = 1$	16	8
②	$\frac{-1}{2}(3)(1) = -1.5$	$1 + \frac{2}{3}(3) = 3$	$1 + \frac{2}{3}(1) = 1.67$	-4.5	-2.505
③	$\frac{-1}{4}\pi(1)^2 = -0.785$	$\frac{4}{3\pi}(1) = 0.424$	0.424	-0.333	-0.333
	5.715			11.167	5.162

$$\bar{x} = \frac{11.167}{5.715} = \boxed{1.95 \text{ in}}$$

$$\bar{y} = \frac{5.162}{5.715} = \boxed{0.903 \text{ in}}$$

Problem 10.22

Determine I_x



$$I_x = \int_A \tilde{y}^2 dA$$

$$\tilde{y} = y$$

$$dA = x\,dy$$

$$x = \frac{y^2}{2}$$

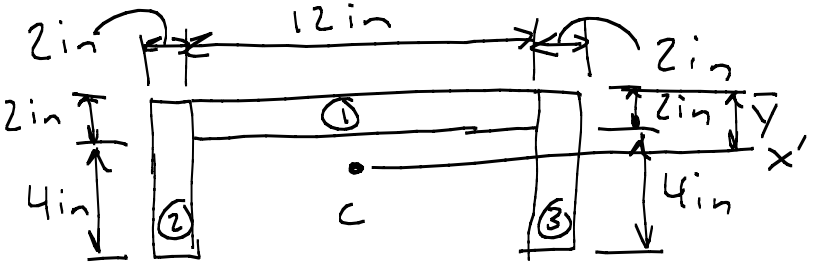
$$I_x = \int_0^2 y^2 x\,dy$$

$$I_x = \int_0^2 y^2\left(\frac{y^2}{2}\right)dy = \frac{1}{2}\int_0^2 y^4 dy = \frac{1}{2}\left(\frac{1}{5}y^5\right)\Big]_0^2$$

$$\boxed{I_x = 3.2\,\text{in}^4}$$

Problem 10.45

Determine: $\bar{y}$ and $I_{x'}$

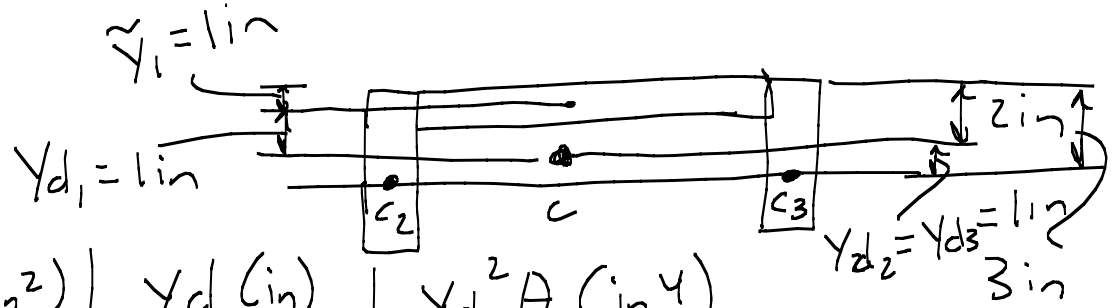


$\bar{Y}$

Shape	$A(in^2)$	$\tilde{y}$ (in) from top	$\tilde{y}A(in^3)$
①	$(12)(2) = 24$	$\frac{1}{2}(2) = 1$	24
②	$(2)(6) = 12$	$\frac{1}{2}(6) = 3$	36
③	12	3	36
	48		96

$$\bar{Y} = \frac{96}{48} = \boxed{2in}$$

$I_{x'}$



Shape	$I_{\tilde{x}}$ (in^4)	A (in^2)	y_d (in)	$y_d^2 A$ (in^4)
①	$\frac{1}{12}(12)(2)^3 = 8$	24	1	24
②	$\frac{1}{12}(2)(6)^3 = 36$	12	1	12
③	36	12	1	12
	80			48

$$I_{x'} = 80 + 48 = \boxed{128\ in^4}$$