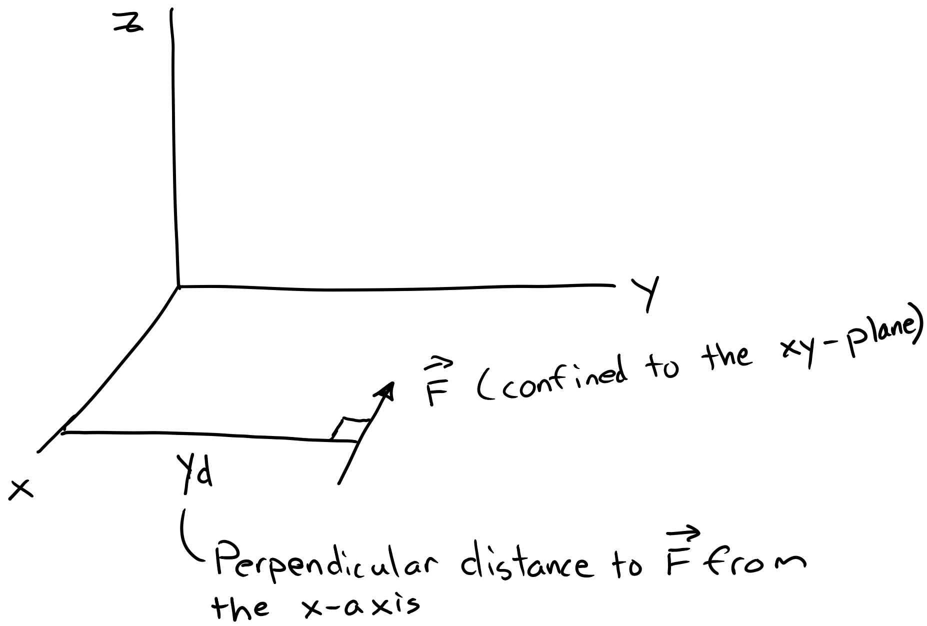
4.1) Moments of Force

Moment of Force

- Force about a point or axis that produces a tendency to rotate about that point or axis
- Force and Moment (Lever) Arm



Magnitude of the Moment

$$M = F y_d$$

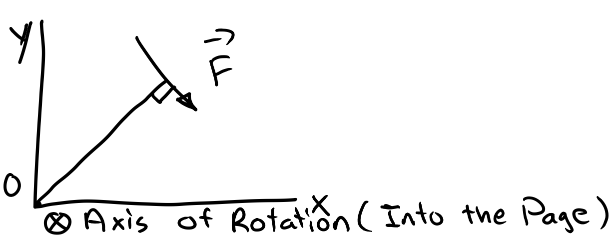
Moment of Force

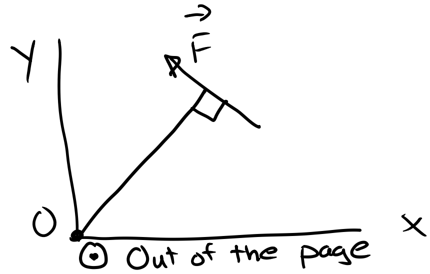
In General,

$$M = F d_{\perp}$$

Direction

- Axis of Rotation
- Found using the Right-Hand Rule
    - Fingers form the moment arm
    - Fingers curl in the direction of the force
    - Thumbs points along the positive axis of rotation

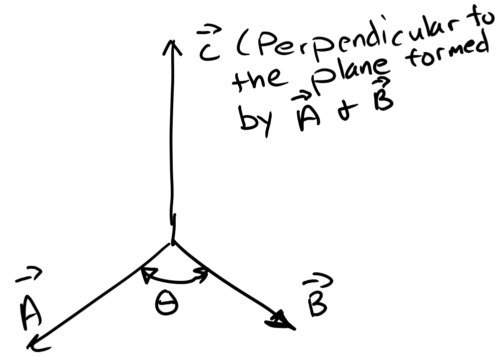




Cross-Product
- Best method for finding moments of force in 3-D

$$\vec{C} = \vec{A} \times \vec{B}$$

(Cross-Product)



Magnitude

$$C = AB\sin\theta$$

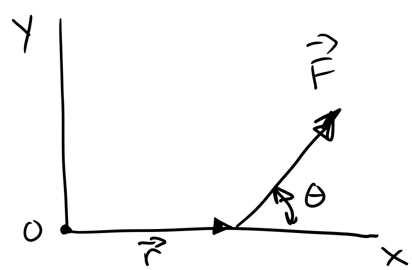
Direction
- Perpendicular to the plane formed by **A** and **B**

$$\vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta)\,\vec{u}_c$$

$$M_o = (F\sin\theta)\,r$$

same

$$\vec{r} \times \vec{F} = ((r)(F)\sin\theta)\,\hat{k}$$



$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

$$\boxed{\vec{A} \times \vec{B} = \{(A_yB_z - A_zB_y)\hat{\imath} - (A_xB_z - A_zB_x)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}\}}$$

Note $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Could also use the determinant to calculate the cross-product

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \rightarrow \hat{i}(A_yB_z - A_zB_y)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \rightarrow -\hat{j}(A_xB_z - A_zB_x)$$

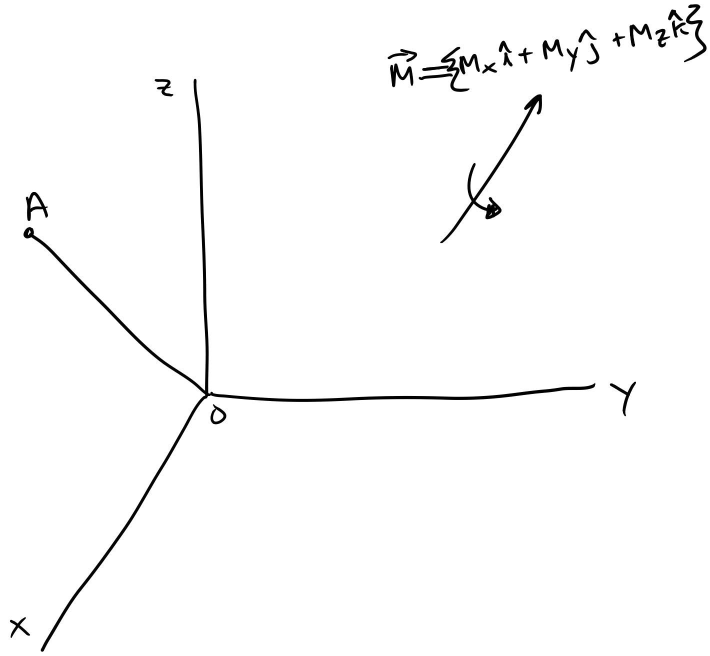
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \rightarrow \hat{k}(A_xB_y - A_yB_x)$$

$$\boxed{\vec{A} \times \vec{B} = \{(A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}\}}$$

Moment of Force about a Specific Axis

$$M_{OA} = \vec{M} \cdot \vec{u}_{OA}$$

$$\vec{M}_{OA} = M_{OA}\vec{u}_{OA}$$



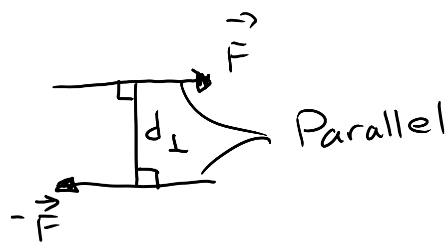
Could also use the Triple Scalar Product
- Incorporates the Dot-Product and Cross-Product into a single operation

$$M_{OA} = \left(\vec{u}_{OA} \cdot \vec{M}\right) = \vec{u}_{OA} \cdot \left(\vec{r} \times \vec{F}\right)$$

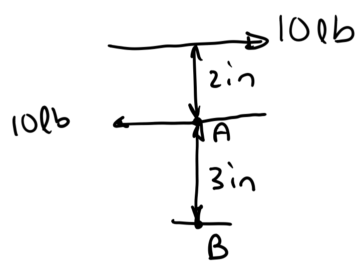
$$M_{OA} = \begin{vmatrix} u_{OAx} & u_{OAy} & u_{OAz} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \quad u_{OAx}\overbrace{\left(r_y F_z - r_z F_y\right)}^{M_x} - u_{OAy}\left(r_x F_z - r_z F_x\right) + u_{OAz}\left(r_x F_y - r_y F_x\right)$$

Couple Moments

Force Couple



Resultant Moment (Couple Moment)
- Found by summing moments about any point



$$\circlearrowright + \; \Sigma M_A \Rightarrow (10\,lb)(2\,in) = 20\,lb\cdot in$$

$$\circlearrowright + \; \Sigma M_B \Rightarrow -(10\,lb)(3\,in) + (10\,lb)(5\,in) = 20\,lb\cdot in$$

Couple moments are **Free Vectors** (can be applied anywhere on the system)