ENGR 2241 – Statics
Section 5: Equilibrium of a Rigid Body

Internal versus External Forces

Internal Forces: Interaction of adjacent particles within a body
Sum of Internal Forces = 0 (Newton's 3rd Law)

External Forces: Outside of the body

Free-Body Diagrams show only external forces

Static Equilibrium

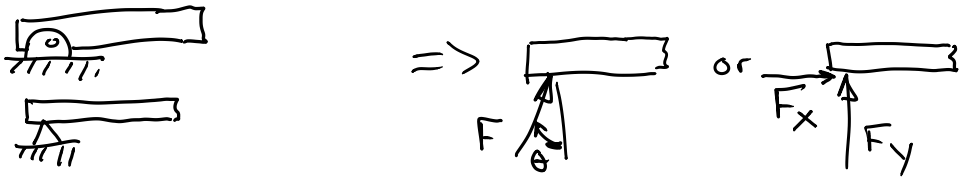
$$\Sigma \vec{F} = 0$$
$$\Sigma \vec{M} = 0$$

Support Reactions in 2-D
- If the support reaction prevents translation, then a force is developed in the opposite direction of the intended translation
- If the support reaction prevents rotation, then a couple moment is developed in the opposite direction of the intended rotation

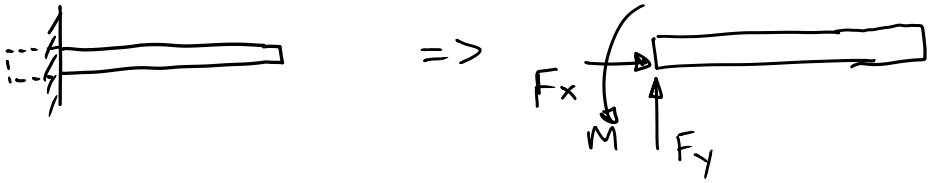
Examples

Roller Support

Pinned Support



Fixed Support

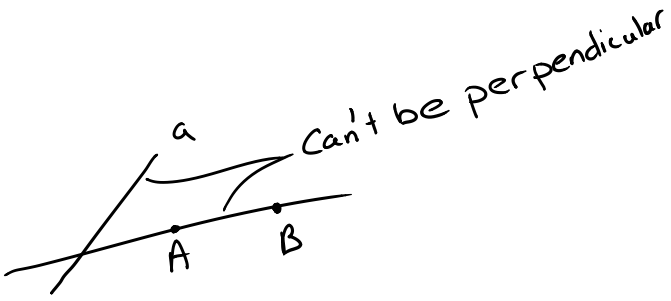


Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

or

$$\Sigma F_a = 0$$
$$\Sigma M_A = 0$$
$$\Sigma M_B = 0$$

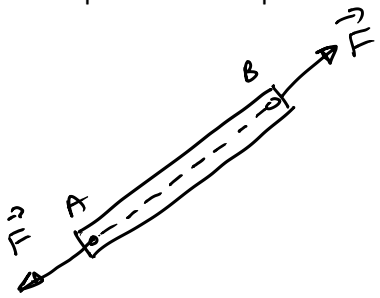


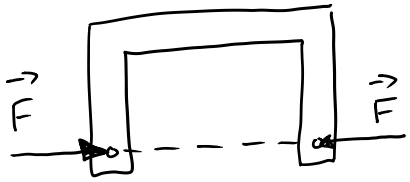
or

$$\Sigma M_A = 0$$
$$\Sigma M_B = 0$$
$$\Sigma M_C = 0$$

A, B, & C can't be on the same line

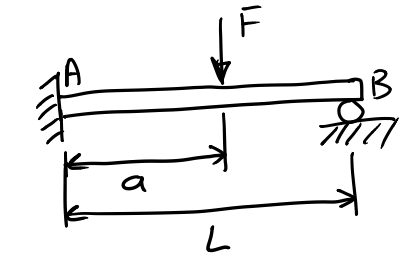
Two-Force Members
- Forces are applied only at two points
- No couple moment is produced

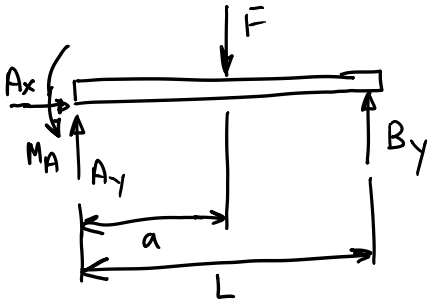




Statically Indeterminate Structures

- More support reactions than equilibrium equations





$\pm \Sigma F_x = 0 \Rightarrow A_x = 0$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y + B_y - F = 0$

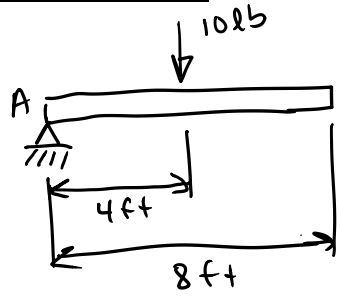
$+\circlearrowleft \Sigma M_A = 0 \Rightarrow M_A - Fa + B_y L = 0$

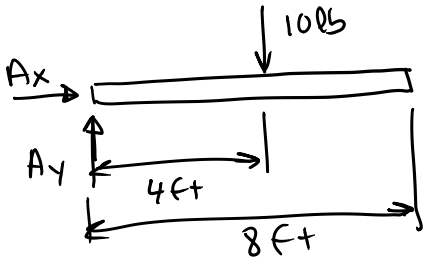
3 Equations, 4 unknowns $\Rightarrow$ Statically Indeterminate

$\Rightarrow$ Can't be solved using Equilibrium alone

$\Rightarrow$ Must look at deformations

Unstable Structures





$\pm \Sigma F_x = 0 \Rightarrow \boxed{A_x = 0}$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y - 10lb = 0$

$\boxed{A_y = 10lb}$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow -(10lb)(4ft) = 0$

$-40 = 0 \Rightarrow$ Unstable

Support Reactions in 3-D

- Translation in the x, y, and z directions
- Rotation about the x, y, and z axes

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$