Moment of Inertia

First Moment of Area

$$= \int x \, dA$$

Second Moment of Area

- Called the Moment of Inertia (I)

$$I = \int \tilde{x}^2 dA$$

$$I_x = \int \tilde{y}^2 dA \quad \text{(Moment of Inertia about the x-axis)}$$

$$I_y = \int \tilde{x}^2 dA \quad \text{(Moment of Inertia about the y-axis)}$$

Polar Moment of Inertia (J)

$$J = \int r^2 dA = \int (\tilde{x}^2 + \tilde{y}^2) dA$$

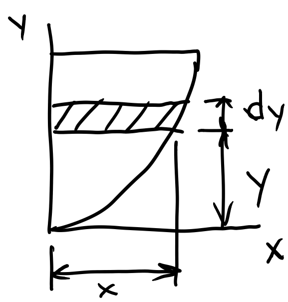
$$= \int \tilde{x}^2 dA + \int \tilde{y}^2 dA$$

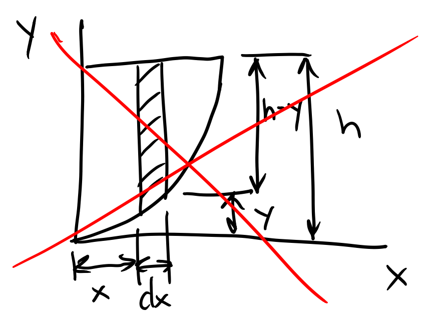
$$\boxed{J = I_y + I_x}$$

Use of differential strips

- Strip must be parallel to the axis about which you are finding the moment of inertia

Find I_x





$I_{rectangle} = \frac{1}{12} b h^3$

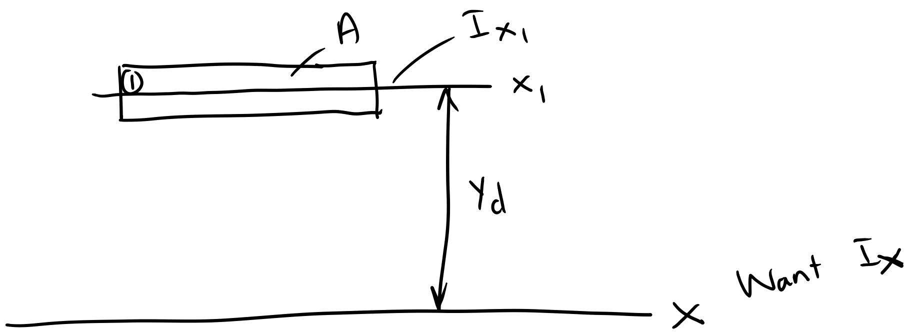
$I_{strip} = \frac{1}{12} x \, dy^3$ → dy^3 = Higher Order Term (Neglect)

$I_{strip} = \frac{1}{12} dx (h-y)^3$

(Not a Higher Order Term => Can't neglect)

<u>Parallel-Axis Theorem</u>

- Allows for the moment of inertia to be transformed from one axis to another
- Used for composite shapes



$$I_x = I_{x_1} + y_d^2 A$$

<u>In General</u>

Any Axis

$$I_{x'} = \Sigma \left(I_{x_i} + y_{d_i}^2 A_i \right)$$

$$I_{y'} = \Sigma \left(I_{y_i} + x_{d_i}^2 A_i \right)$$