

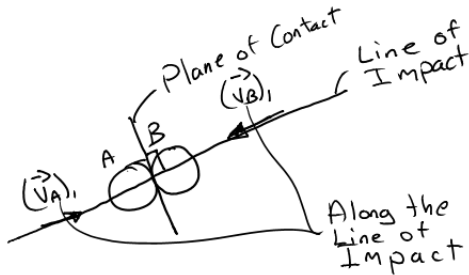
Impact

- Collision between two bodies during a short time duration

Types of Impact

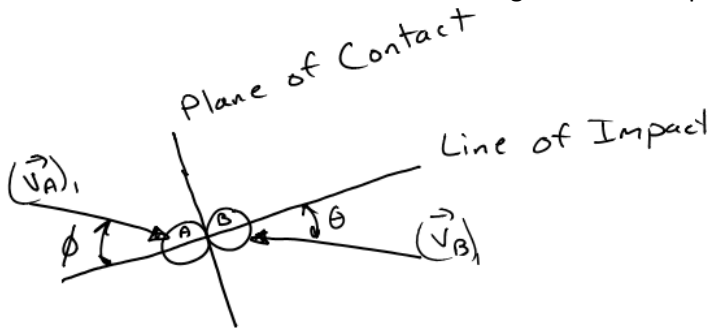
- Central Impact

- The initial velocities are directed along the line of impact



- Oblique Impact

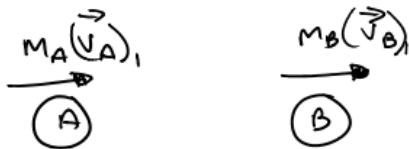
- The initial velocities are not directed along the line of impact



- For both types of impact, all impulses are assumed to be directed along the line of impact

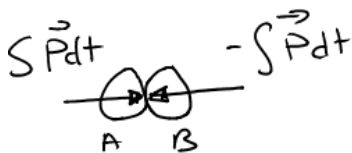
Stages of Impact

1. Before Impact



2. Deformation Impulse

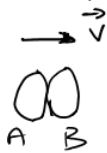
- Collision occurs and the particles deform



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Impact

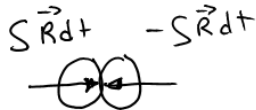
3. Maximum Deformation

- Relative deformation is zero
- Both particles move with the same velocity



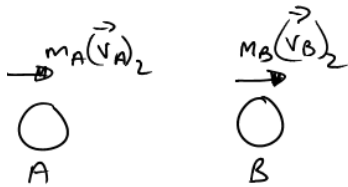
4. Period of Restitution

- The particles begin to return to their original shape or remain completely deformed



$$\int P dt > \int R dt$$

5. After Impact



- Apply the Conservation of Momentum for the system of particles (A and B) between 1 and 5

$$\rightarrow m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2$$

- Apply the Principle of Linear Impulse and Momentum for Particle A between 1 and 3

$$\rightarrow m_A (v_A)_1 - \int P dt = m_A v$$

- Apply the Principle of Linear Impulse and Momentum for Particle A between 3 and 5

$$\rightarrow m_A v - \int R dt = m_A (v_A)_2$$

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- Define the Coefficient of Restitution, e
 - Determined experimentally
 - For an elastic impact: $e = 1$
 - For a plastic impact: $e = 0$

$$e = \frac{\int R dt}{\int P dt} = \frac{m_A v - m_A (v_A)_2}{m_A (v_A)_1 - m_A v}$$

$$e = \frac{v - (v_A)_2}{(v_A)_1 - v}$$

- Repeat for Particle B

$$m_B (v_B)_1 + \int P dt = m_B v$$
$$m_B v + \int R dt = m_B (v_B)_2$$

$$e = \frac{(v_B)_2 - v}{v - (v_B)_1}$$

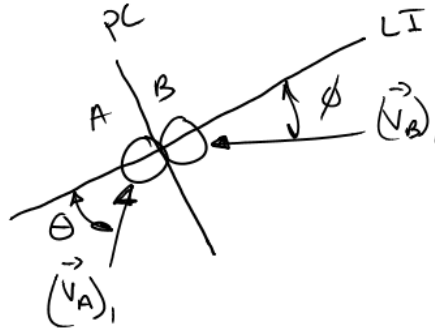
- Eliminate v using the two expressions for e

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

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- For an oblique impact:

- Apply the Conservation of Momentum for the system of particles and coefficient of restitution equations along the line of impact



$$\rightarrow + m_A (v_A)_{1,LI} + m_B (v_B)_{1,LI} = m_A (v_A)_{2,LI} + m_B (v_B)_{2,LI}$$

$$\rightarrow + e = \frac{(v_B)_{2,LI} - (v_A)_{2,LI}}{(v_A)_{1,LI} - (v_B)_{1,LI}}$$

- Apply the Conservation of Momentum for each particle along the plane of contact

$$\rightarrow + m_A (v_A)_{1,PC} = m_A (v_A)_{2,PC}$$

$$\boxed{(v_A)_{1,PC} = (v_A)_{2,PC}}$$

$$\rightarrow + m_B (v_B)_{1,PC} = m_B (v_B)_{2,PC}$$

$$\boxed{(v_B)_{1,PC} = (v_B)_{2,PC}}$$