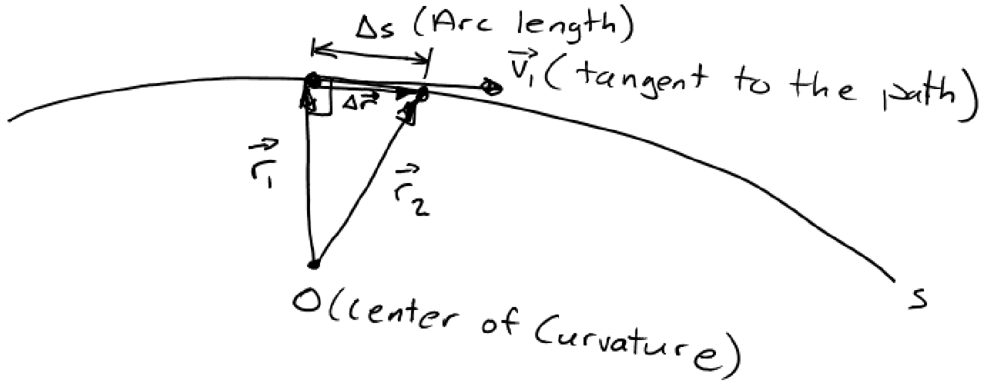
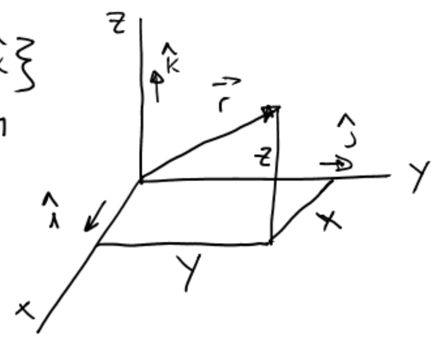


<u>Curvilinear Motion: Rectangular Components</u>

- Position

$$\vec{r} = \{x\hat{i} + y\hat{j} + z\hat{k}\}$$

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$



<u>Direction of $\vec{r}$</u>

$$\vec{u}_r = \frac{\vec{r}}{r}$$

- Velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\vec{v} = \left\{\underbrace{\frac{dx}{dt}}_{v_x}\hat{i} + \underbrace{\frac{dy}{dt}}_{v_y}\hat{j} + \underbrace{\frac{dz}{dt}}_{v_z}\hat{k}\right\}$$

<u>Alternate Terminology</u>

$v_x = \frac{dx}{dt} = \dot{x}$ $\quad \bullet - \frac{d}{dt}$ $\qquad v_y = \frac{dy}{dt} = \dot{y} \qquad v_z = \frac{dz}{dt} = \dot{z}$

- Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right)$$

$$= \left\{\underbrace{\frac{dv_x}{dt}}_{a_x}\hat{i} + \underbrace{\frac{dv_y}{dt}}_{a_y}\hat{j} + \underbrace{\frac{dv_z}{dt}}_{a_z}\hat{k}\right\}$$

$$a_x = \frac{dv_x}{dt} = \dot{v}_x = \ddot{x} \qquad a_y = \dot{v}_y = \ddot{y} \qquad a_z = \dot{v}_z = \ddot{z}$$

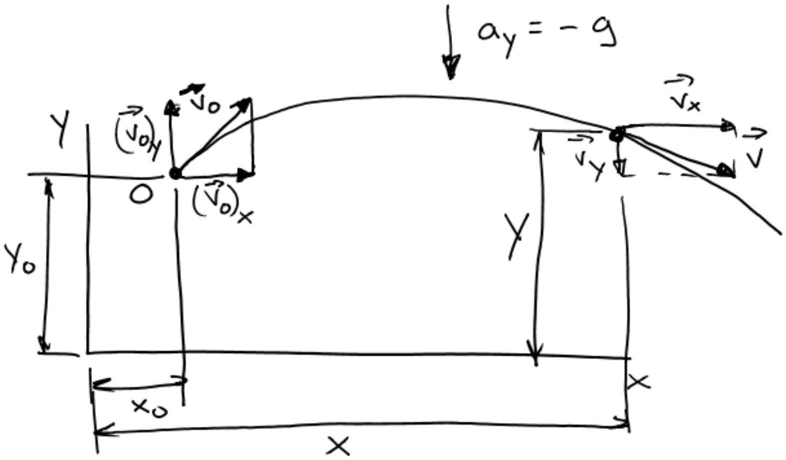
Projectile Motion

- Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$



- Horizontal Motion ($a_x = 0$)

$$\xrightarrow{+} \quad v_x^2 = (v_0)_x^2 \quad = (v_x) = (v_0)_x$$

$$\xrightarrow{+} \quad v_x = (v_0)_x$$

$$\xrightarrow{+} \quad \boxed{x = x_0 + (v_0)_x t}$$

- Vertical Motion ($a_y = -g$)

$$+\uparrow \quad v_y^2 = (v_0)_y^2 + 2(-g)(y - y_0)$$

$$\boxed{v_y^2 = (v_0)_y^2 - 2g(y - y_0)}$$

$$+\uparrow \quad v_y = (v_0)_y + (-g)t$$

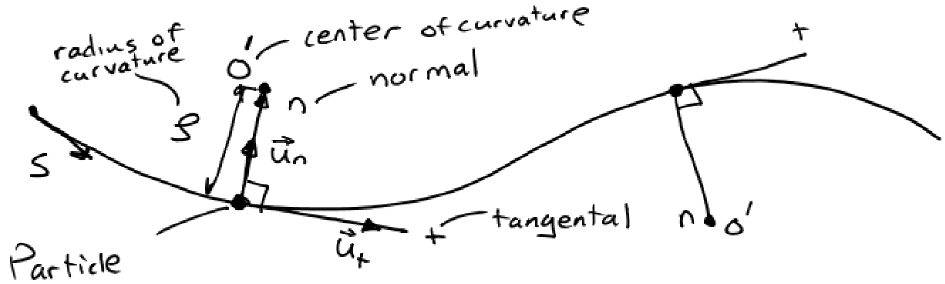
$$\boxed{v_y = (v_0)_y - gt}$$

$$+\uparrow \quad y = y_0 + (v_0)_y t + \frac{1}{2}(-g)t^2$$

$$\boxed{y = y_0 + (v_0)_y t - \frac{1}{2}gt^2}$$

Normal and Tangental Coordinates

- Alternate coordinate system



- Velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\boxed{\vec{v} = v\,\vec{u}_t}$$

velocity is always tangent to the path

- Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v\,\vec{u}_t\right)$$

$$= \underbrace{\frac{dv}{dt}}_{a_t}\vec{u}_t + v\frac{d\vec{u}_t}{dt}$$

ASM $\quad \frac{d\vec{u}_t}{dt} = \frac{v}{\rho}\vec{u}_n$

$$\boxed{a_t = \frac{dv}{dt}} \qquad \boxed{a_n = \frac{v^2}{\rho}}$$

$$\boxed{\vec{a} = a_t\vec{u}_t + a_n\vec{u}_n}$$

$$\boxed{a = \sqrt{a_t^2 + a_n^2}}$$

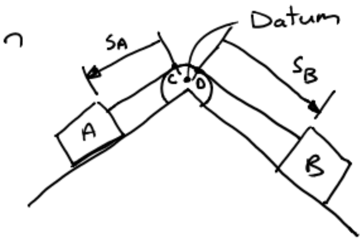
$$\boxed{a_t ds = v dv}$$

- For a non-circular path:

$$\boxed{\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}}$$

Dependent Motion

Define the position of each block with respect to a fixed point (Datum)



– Position => Positive away from the datum

Total Length, l_T

$$\underbrace{l_T}_{\text{cable}} = s_A + s_B + l_{CD}$$

Differentiate with respect to time

$$\frac{\cancel{d l_T}^{\,0}}{\cancel{dt}} = \frac{ds_A}{dt} + \frac{ds_B}{dt} + \frac{\cancel{d l_{CD}}^{\,0}}{\cancel{dt}}$$

l_T & l_{CD} => constant

$$0 = v_A + v_B \qquad \boxed{v_A = -v_B}$$

Differentiate Again

$$0 = a_A + a_B \qquad \boxed{a_A = -a_B}$$

Typically

Group l_{CD} with l_T

$$l = l_T - l_{CD}$$