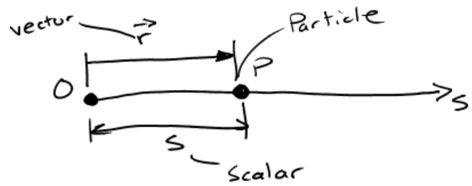
Kinematics of a Particle – Kinematics for Rectilinear Motion

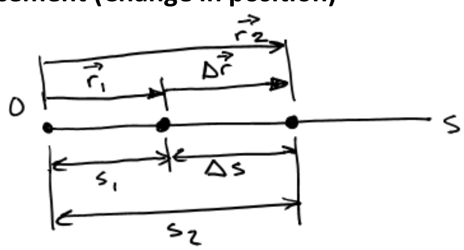
Terminology
- **Dynamics**: Accelerated motion of a body
- **Kinematics**: Geometric aspects of motion
- **Kinetics**: Analysis of forces that cause motion
- **Particle:** Has mass, but no geometry impact (neglect the impact of the shape of the body)

Rectilinear Kinematics of a Particle
- Rectilinear: Straight-line path

- Position



- Displacement (Change in position)



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$\Delta s = s_2 - s_1$$

- Average Velocity

Average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad v_{avg} = \frac{\Delta s}{\Delta t}$$

- Average Speed

Total Distance traveled

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

- Instantaneous Velocity

$$\vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{r}}{\Delta t}\right) \Rightarrow \boxed{\vec{v} = \frac{d\vec{r}}{dt}}$$

$$\boxed{v = \frac{ds}{dt}}$$

- Average Acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

- Instantaneous Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\boxed{a = \frac{dv}{dt}}$$

- Kinematic Relation between Acceleration, Velocity, and Position

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$dt = \frac{ds}{v} \qquad dt = \frac{dv}{a}$$

$$\frac{ds}{v} = \frac{dv}{a}$$

$$\boxed{a\,ds = v\,dv}$$

- The Key Kinematic Relationships

$$\boxed{a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \qquad a\,ds = v\,dv}$$

- Special Case: Constant Acceleration

$$a = a_c$$

$$a = \frac{dv}{dt}$$

$$\int_0^t a_c\,dt = \int_{v_0}^{v} dv$$

$$a_c \int_0^t dt = \int_{v_0}^{v} dv$$

$$a_c t\Big]_0^t = v\Big]_{v_0}^{v}$$

$$a_c t = v - v_0$$

$$\boxed{v = v_0 + a_c t}$$

$$v = \frac{ds}{dt}$$

$$\int_0^t v\,dt = \int_{s_0}^{s} ds$$

$$v = v_0 + a_c t$$

$$\int_0^t (v_0 + a_c t)\,dt = \int_{s_0}^{s} ds$$

$$\left(v_0 t + \frac{1}{2} a_c t^2\right)\Big]_0^t = s\Big]_{s_0}^{s}$$

$$v_0 t + \frac{1}{2} a_c t^2 = s - s_0$$

$$\boxed{s = s_0 + v_0 t + \frac{1}{2} a_c t^2}$$

$$a\,ds = v\,dv$$

$$\int_{s_0}^{s} a_c\,ds = \int_{v_0}^{v} v\,dv$$

$$a_c(s)\Big]_{s_0}^{s} = \frac{1}{2} v^2\Big]_{v_0}^{v} \Rightarrow a_c(s - s_0) = \frac{1}{2} v^2 - \frac{1}{2} v_0^2$$

$$\boxed{v^2 = v_0^2 + 2a_c(s - s_0)}$$