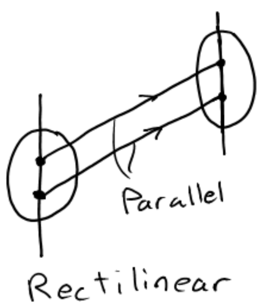
ENGR 2242 – Dynamics
Planar Kinematics of a Rigid Body

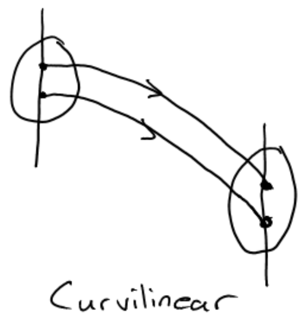
Planar Motion
- All motion is confined to a 2-D plane

Rigid Body
- The geometry of the body is taken into account
- No deformation

Types of Planar Motion

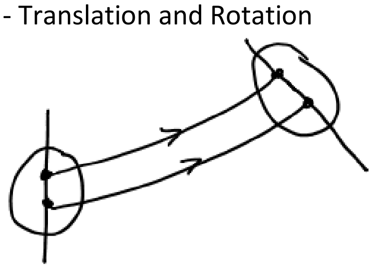
-Translation





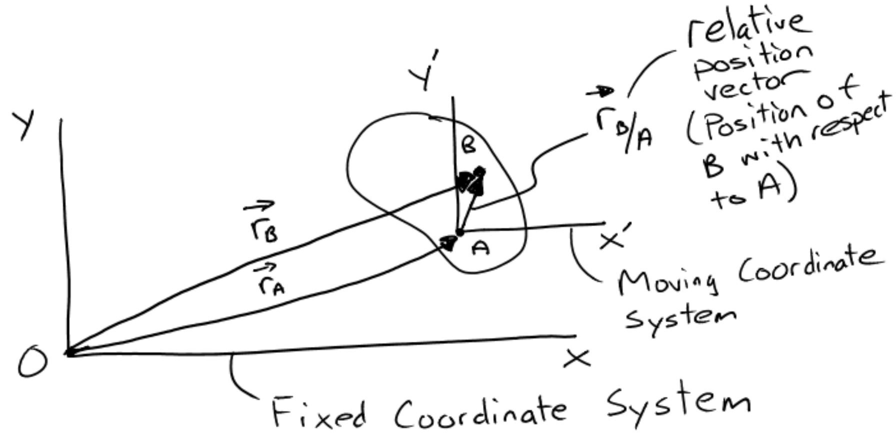
- Rotation about a Fixed Axis

- General Plane Motion
    - Translation and Rotation



Kinematics of a Rigid Body

- Translation



- Position

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

- Velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \cancel{\frac{d\vec{r}_{B/A}}{dt}} \xrightarrow{} 0$$

$\vec{r}_{B/A}$ does not change in magnitude or direction for translation

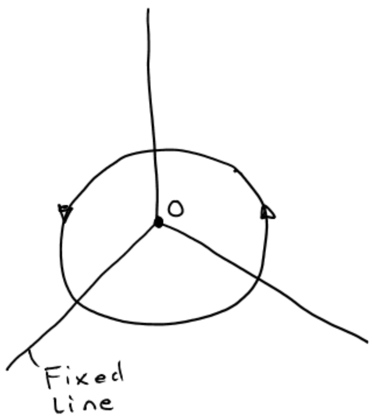
$$\boxed{\vec{v}_B = \vec{v}_A}$$

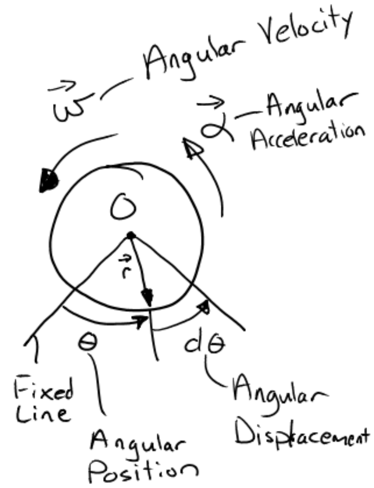
- Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\boxed{\vec{a}_B = \vec{a}_A}$$

- Rotation about a Fixed Axis
    - Use angular quantities





- Angular Velocity

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

- Angular Acceleration

$$\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}}$$

$$\alpha = \frac{d\omega}{dt} \qquad \omega = \frac{d\theta}{dt}$$

$$dt = \frac{d\omega}{\alpha} \qquad dt = \frac{d\theta}{\omega}$$

$$\frac{d\omega}{\alpha} = \frac{d\theta}{\omega}$$

$$\boxed{\alpha\, d\theta = \omega\, d\omega}$$

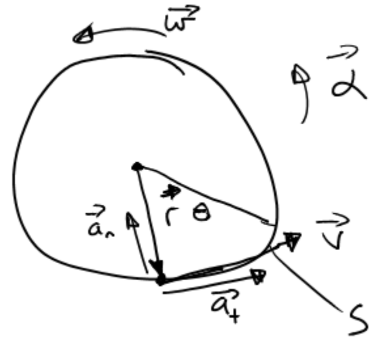
- Constant Angular Acceleration

$$\omega = \omega_0 + \alpha_c t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

- Relating Angular Kinematics to Non-angular Kinematics



Arc Length Formula

$$s = r\theta$$

$$v = \frac{ds}{dt}$$

$$= \frac{d}{dt}(r\theta)$$

$$= r\frac{d}{dt}(\theta)$$

$$v = r\frac{d\theta}{dt}$$

$$\boxed{v = \omega r}$$

In Vector Form

$$\boxed{\vec{v} = \underbrace{\vec{\omega}}_{\hat{k}} \times \underbrace{\vec{r}}_{\hat{i}\ \&\ \hat{j}\text{ components}}}$$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(\omega r)$$

$$= r\frac{d\omega}{dt}$$

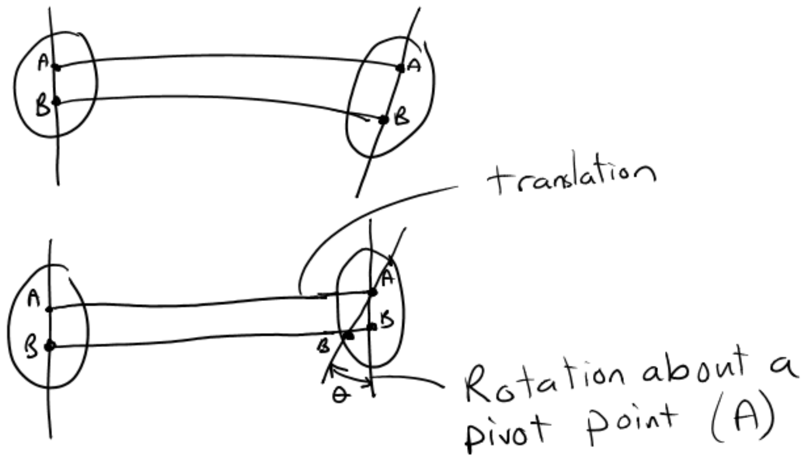
$$\boxed{a_t = \alpha r} \qquad \boxed{\vec{a}_t = \vec{\alpha} \times \vec{r}}$$

$$a_n = \frac{v}{\rho}$$

$$= \frac{(\omega r)^2}{r}$$

$$\boxed{a_n = \omega^2 r} \qquad \boxed{\vec{a}_n = -\omega^2 \vec{r}}$$

- General Plane Motion



- Position

$$\vec{r} = \vec{r}_A + \vec{r}_{B/A}$$

- Velocity

$\frac{d\vec{r}_{B/A}}{dt}$ is not zero for general plane motion

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}}$$

- Acceleration

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$

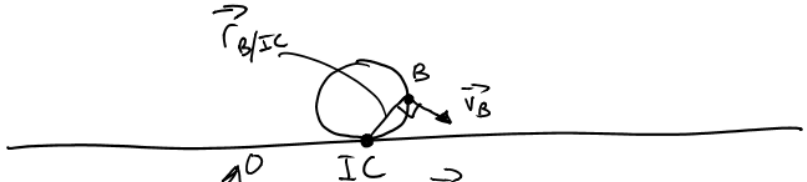
$$\left(\vec{a}_{B/A}\right)_t = \vec{\alpha}_{AB} \times \vec{r}_{B/A}$$

$$\left(\vec{a}_{B/A}\right)_n = -\omega_{AB}^2 \vec{r}_{B/A}$$

$$\boxed{\vec{a}_B = \vec{a}_A + \left(\vec{\alpha}_{AB} \times \vec{r}_{B/A}\right) - \omega_{AB}^2 \vec{r}_{B/A}}$$

The Instantaneous Center of Zero Velocity (IC)

- At any time of motion, there is always a point where all points on the rigid body pivot about
- At that instant, that point has zero translational velocity
- Locating the IC can be done by inspection or by using geometry
    - Scenario 1



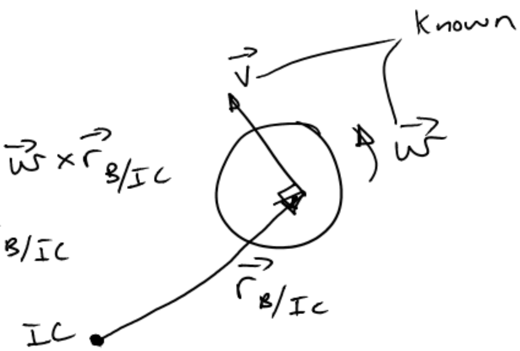
$$\vec{v}_B = \cancel{\vec{v}_{IC}}^{\,0} + \vec{\omega} \times \vec{r}_{B/IC}$$

- Scenario 2

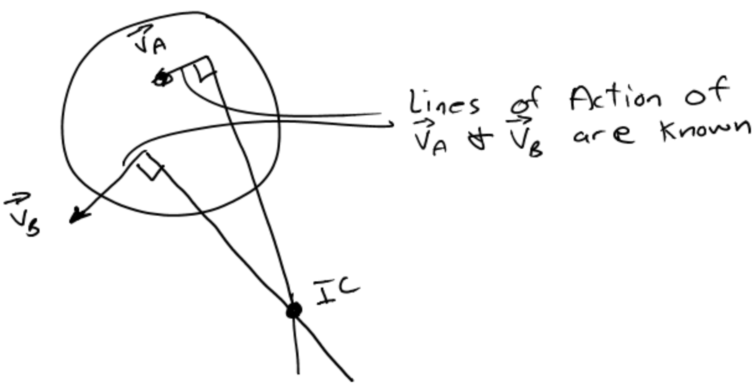
$$\vec{v} = \cancel{\vec{v}_{IC}}^{\,0} + \vec{\omega} \times \vec{r}_{B/IC}$$

$$v = \omega\, r_{B/IC}$$

$$\boxed{r_{B/IC} = \frac{v}{\omega}}$$



- Scenario 3



- Scenario 4