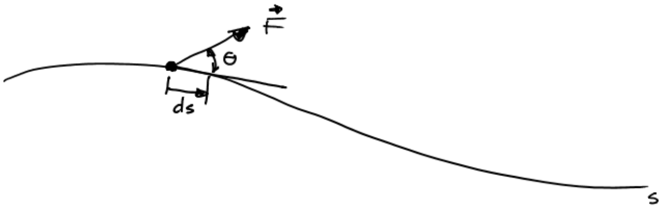
ENGR 2242 – Dynamics
Kinetics of a Particle – Principle of Work and Energy

Work

- A force does work only when the particle undergoes a displacement in the direction of the force
- Work = Force x Displacement in the Direction of the Force
- Work is positive if the direction of the force is the same as the direction of the displacement, and negative if opposite



- Expression for Work

$$\underbrace{dU}_{\text{Work}} = (F\cos\theta)\,ds$$

$$\boxed{U = \int_{s_1}^{s_2} (F\cos\theta)\,ds}$$

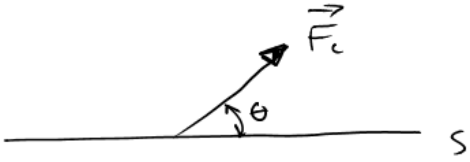
$$dU = F\,ds\cos\theta$$

Dot Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$dU = \vec{F}\cdot d\vec{r}$$

- Work of a Constant Force

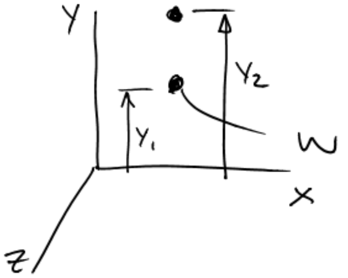


$$U_{1-2} = \int_{s_1}^{s_2} F_c\cos\theta\,ds$$

$$U_{1-2} = F_c\cos\theta\int_{s_1}^{s_2} ds$$

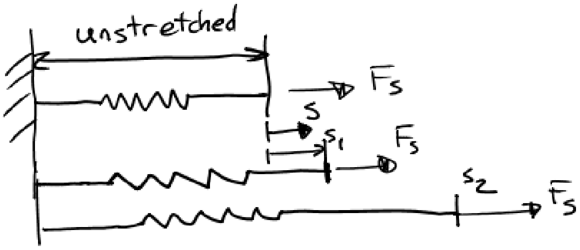
$$\boxed{U_{1-2} = F_c\cos\theta\,(s_2 - s_1)}$$

- Work of a Weight



$$\boxed{U_{1-2} = -W(y_2 - y_1)}$$

- Work of a Spring



$$F_s = \overset{\text{Spring Constant}}{k} s$$

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds$$

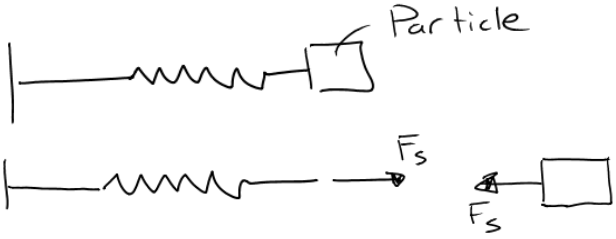
$$= \int_{s_1}^{s_2} ks \, ds$$

$$= k \int_{s_1}^{s_2} s \, ds$$

$$U_{1-2} = \frac{1}{2} ks^2 \Big]_{s_1}^{s_2}$$

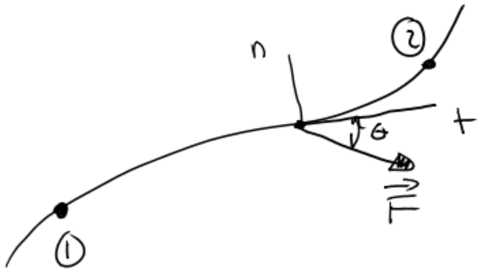
$$\boxed{U_{1-2} = \frac{1}{2} k \left(s_2^2 - s_1^2 \right)}$$

- For a Particle attached to the Spring



$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$

Principle of Work and Energy



$$\Sigma F_t = ma_t$$

$$a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$$

$$v = \frac{ds}{dt}$$

$$a_t\,ds = v\,dv$$

$$a_t = v\frac{dv}{ds}$$

$$\Sigma F_t = m\left(v\frac{dv}{ds}\right)$$

$$\Sigma F_t\,ds = mv\,dv$$

$$\Sigma \int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\underbrace{\Sigma \int_{s_1}^{s_2} F\cos\theta\,ds}_{U_{1-2}} = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$\Sigma U_{1-2} = \underbrace{\tfrac{1}{2}mv_2^2}_{\text{Kinetic Energy}(T)} - \tfrac{1}{2}mv_1^2$$

$$\boxed{T_1 + \Sigma U_{1-2} = T_2}$$

Principle of Work & Energy

- For a System of Particles

System of Particles

$$\sum \frac{1}{2} m_i (v_i)_1^2 + \underbrace{\sum \int_{s_1}^{s_2} (F_i)_t \, ds}_{\text{External Forces}} + \underbrace{\sum \int_{s_1}^{s_2} (f_i)_t \, ds}_{\text{Internal Forces}} = \sum \frac{1}{2} m_i (v_i)_2^2$$

not always zero, Forces are equal and opposite, but the paths are not always the same

- If there is translation only, no particle deformation, and particles are connected using rigid cables:

$$\boxed{\sum T_1 + \underbrace{\sum U_{1-2}}_{\text{External Forces only}} = \sum T_2}$$

Power

$$P = \frac{dU}{dt}$$

$$P = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}}$$

$$\boxed{P = \vec{F} \cdot \vec{v}}$$