Principal Stresses

- Maximum and minimum normal stress
- For a maximum or minimum

$$\frac{d\sigma_{x_1}}{d\theta} = 0$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\frac{d\sigma_{x_1}}{d\theta} = 0 - \left(\frac{\sigma_x - \sigma_y}{2}\right) 2\sin 2\theta_p + (\tau_{xy})(2\cos 2\theta_p) = 0$$

$$\boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

θ_p = Angle that locates the principal stresses

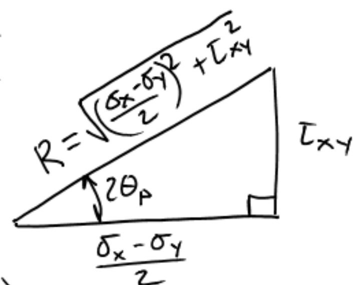
$2\theta_p$ => Two values => Differ by 180° $\begin{pmatrix} 0^\circ - 180^\circ \\ 180^\circ - 360^\circ \end{pmatrix}$

θ_p => Differ by 90°

One angle => Max Principal Stress

Other angle => Min Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$



$$\cos 2\theta_p = \frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{R}$$

$$\sin 2\theta_p = \frac{\tau_{xy}}{R}$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\left(\frac{\frac{\sigma_x - \sigma_y}{2}}{R}\right) + \tau_{xy}\left(\frac{\tau_{xy}}{R}\right)$$

σ_1 => Larger Principal Stress

ASA

$$\boxed{\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

σ_2 = Smaller Principal Stress

$\sigma_1, \sigma_2 \Rightarrow 90^\circ$

$$\sigma_{x_1} + \sigma_{y_1} = \sigma_x + \sigma_y$$

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$\boxed{\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

But, $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

You don't know which σ the angle corresponds to.

$\Rightarrow$ - Determine σ_1, σ_2

- Find θ_p
- Sub θ_p into the original σ_{x_1} formula
  $\Rightarrow \sigma_1$ or σ_2

At $\theta_p \Rightarrow \tau_{x_1 y_1} = 0$

<u>Maximum Shear Stress</u>

- Not at the same angle as the principal stresses

Set $\dfrac{d\tau_{x_1 y_1}}{d\theta} = 0$

$$\boxed{\tan \theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} \quad \text{or} \quad \tan 2\theta_s = \frac{-1}{\tan(2\theta_p)}}$$

$$\boxed{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

$$\tau_{min} = -\tau_{max}$$

Which θ_s corresponds to which τ

$\theta_s \Rightarrow$ Original $\tau_{x_1 y_1}$ formula

Corresponding normal stresses

$$\sigma_{x_1} = \sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2}$$