ENGR 2243 – Mechanics of Materials
Axially Loaded Members

- Axial members are loaded in either tension or compression

<u>Derivation of the Axial Member Equation</u>

- Compatibility
    - Relates deformation to strain

$$\varepsilon = \frac{\delta}{L}$$

- Material Law
    - Relates stress to strain

$$\sigma = E\varepsilon$$

- Equilibrium
    - Relates stress to the applied load


$$\sigma = \frac{P}{A}$$

$$\boxed{N = P}$$

- Use compatibility, the material law, and equilibrium to get the axial member equation

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = E\left(\frac{\delta}{L}\right)$$

$$E\left(\frac{\delta}{L}\right) = \frac{P}{A}$$

$$\boxed{\delta = \frac{PL}{EA}}$$

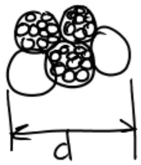
Uniform Stress
Linear Elastic Material

$$P = \underbrace{\left(\frac{EA}{L}\right)}_{\text{Stiffness }(k)}\delta$$

EA ⇒ Axial Rigidity

$$P = k\delta$$

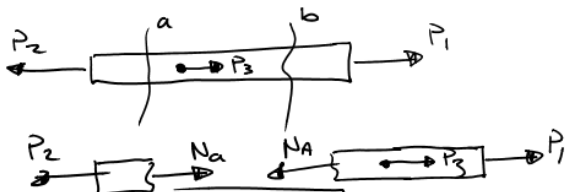
- Steel Cables
    - Transmit tension only
    - Formed by winding small strands of wire



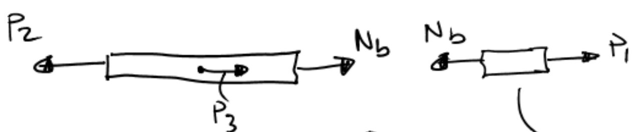

    - Use an effective area (Table 2-1)
    - Also use an effective Modulus of Elasticity (E = 20,000 ksi or 140 GPa)

<u>Non-uniform Bars</u>

- Area and/or load changes along the length

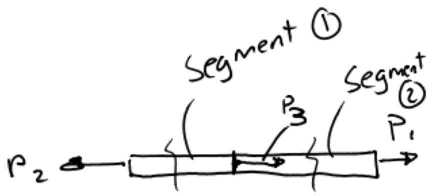



$$\boxed{N_a = P_2}$$



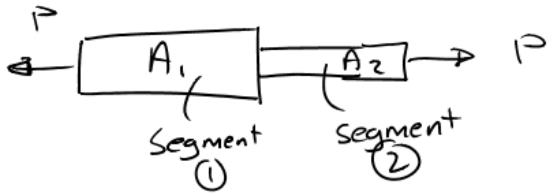

$N_b = P_2 - P_3$ $\qquad$ $N_b = P_1$

- Divide the bar into segments




$$\delta = \underbrace{\delta_1 + \delta_2}_{\text{Watch shortening versus elongating (signs)}}$$

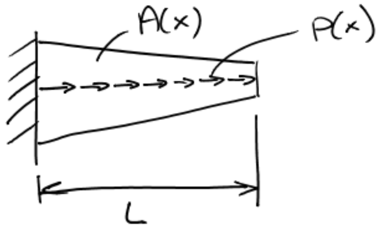
- Another Example




- Divide the bar into segments if the internal force (FBD) and/or the area changes

$$\delta = \sum \frac{PL}{EA}$$

- If the cross-section of the bar varies as a function of the length of the bar, then integrate




$$\delta = \int_0^L \frac{P(x)\,dx}{E\,A(x)}$$