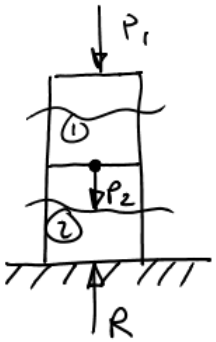
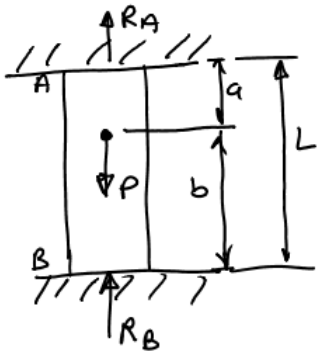
Statically Indeterminate Structures

- Reactions or internal forces can't be determined using equilibrium alone
- Have to look at deformations (Compatibility)



$+\uparrow \Sigma F_y = 0 \Rightarrow -P_1 - P_2 + R = 0$

Statically Determinate



$+\uparrow \Sigma F = -P + R_A + R_B = 0$

$R_A + R_B = P$

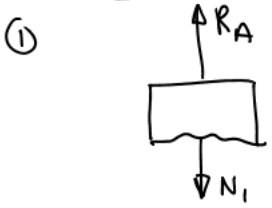
Statically Indeterminate

- Need one more equation
- The Equation of Compatibility
  - The total elongation of the bar is zero

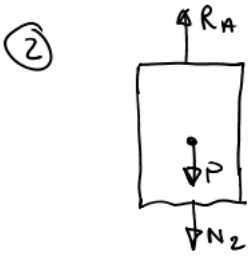
$\delta_{AB} = 0$

- Use equilibrium and compatibility to determine the support reactions

2 segments



$N_1 = R_A$



$+\uparrow \Sigma F = 0 \Rightarrow R_A - P - N_2 = 0$

$N_2 = R_A - P$

$$\delta_1 = \frac{R_A a}{EA} \qquad \delta_2 = \frac{(R_A - P) b}{EA}$$

$$\delta_{AB} = \delta_1 + \delta_2 = 0$$

$$\frac{R_A a}{EA} + \frac{(R_A - P) b}{EA} = 0$$

$$R_A a + R_A b - Pb = 0$$

$$R_A \underbrace{(a+b)}_{=L} - Pb = 0$$

$$\boxed{R_A = \frac{Pb}{L}}$$

$$R_A + R_B = P$$

$$\frac{Pb}{L} + R_B = P$$

$$R_B = P - \frac{Pb}{L}$$

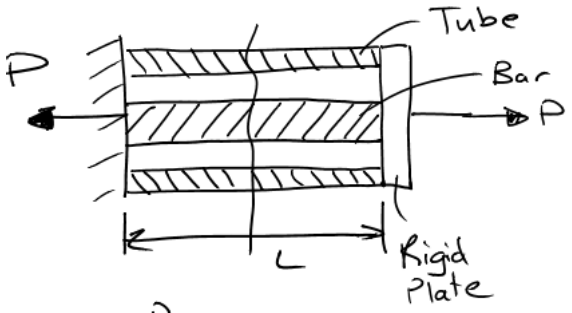
$$R_B = P\left(1 - \frac{b}{L}\right)$$

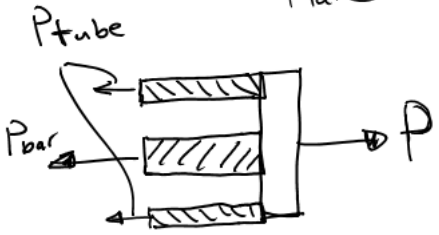
$$R_B = P\left(\frac{L}{L} - \frac{b}{L}\right)$$

$$R_B = P\left(\frac{L-b}{L}\right)$$

$$\boxed{R_B = P\frac{a}{L}}$$

- For a bar inside a tube





$$P = P_{bar} + P_{tube}$$

Compatibility

$$\delta_{bar} = \delta_{tube}$$

$$\frac{P_{bar} L}{E_{bar} A_{bar}} = \frac{P_{tube} L}{E_{tube} A_{tube}}$$