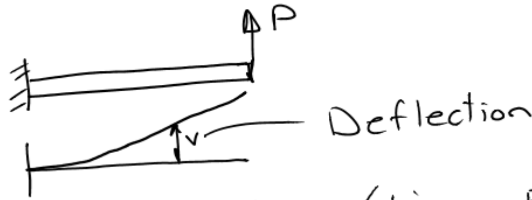
Integration Method



Assume Small Deflections (Linear Elastic Material)

$$M = EI\frac{d^2v}{dx^2}$$ (Differential Equation of the Deflection Curve)

$EI \Rightarrow$ Constant

$$V = \frac{dM}{dx} = EI\frac{d^3v}{dx^3}$$

$$-q = \frac{dV}{dx} = EI\frac{d^4v}{dx^4}$$

Could also use the Prime Convention

$$\boxed{M = EIv'' \quad , \quad V = EIv''' \quad , -q = EIv^{IV}}$$

- Determining Deflections
    - Integrate the moment equation(s) twice
    - For each integration, a constant of integration is generated
        - Use boundary conditions to solve for the constants of integration
            - Deflection (v) or rotation (v') at the supports

Cantilever

$v_A = v(x=0) = 0$

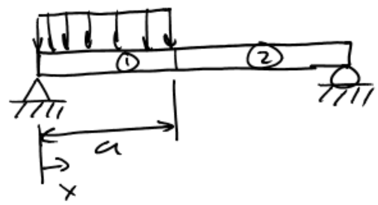
$v'_A = v'(x=0) = 0$

Simply Supported

$v_A = v(x=0) = 0$

$v_B = v(x=L) = 0$

- Multiple Segments



① => $M_1(x)$
Integrate Twice
=> C_1, C_2

② => $M_2(x)$
Integrate Twice
=> C_3, C_4

- 4 Constants of Integration
- Need 4 Boundary Conditions

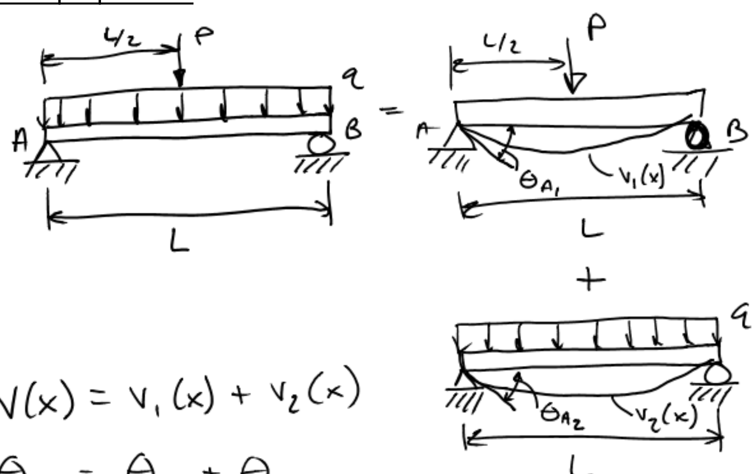
$v_1(x=0) = 0$

$v_2(x=L) = 0$

$v_1(x=a) = v_2(x=a)$

$v_1'(x=a) = v_2'(x=a)$

<u>Method of Superposition</u>



$V(x) = v_1(x) + v_2(x)$

$\theta_A = \theta_{A_1} + \theta_{A_2}$

$R_A = R_{A_1} + R_{A_2}$

- Appendix H: Tables of Beam Deflections for Simply Supported and Cantilever Beams

$$v_1 = -\frac{Px}{48EI}\left(3L^2 - 4x^2\right) \qquad \left(0 \le x \le L/2\right)$$

$$\theta_{A_1} = \frac{PL^2}{16EI}$$

$$v_2 = -\frac{qx}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

$$\theta_{A_2} = \frac{qL^3}{24EI}$$

$$\boxed{\begin{aligned} v(x) &= -\frac{Px}{48EI}\left(3L^2 - 4x^2\right) - \frac{qx}{24EI}\left(L^3 - 2Lx^2 + x^3\right) \\ \theta_A &= \frac{PL^2}{16EI} + \frac{qL^3}{24EI} \end{aligned}}$$

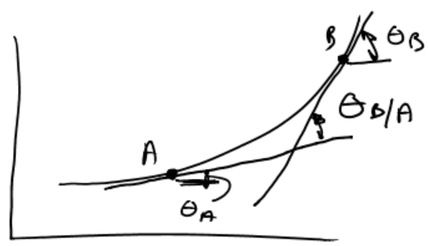
Moment-Area Method

- First Moment-Area Theorem

$$\theta_{B/A} = \theta_B - \theta_A$$

$$M = EIv''$$

$$\theta = v'$$

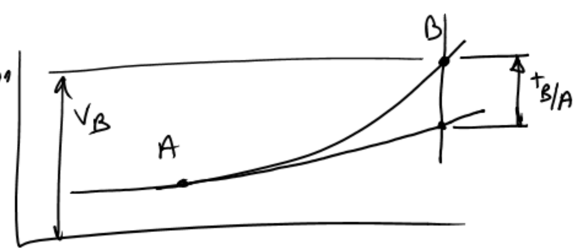


Assume small deflections

$$\theta_{B/A} = \underbrace{\int_A^B \frac{M}{EI}\,dx}_{\text{Area under the } \frac{M}{EI} \text{ diagram between } A \text{ \& } B}$$

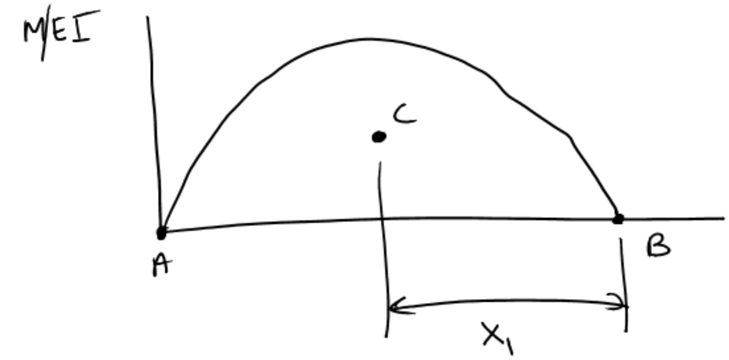
- Second Moment-Area Theorem

$t_{B/A}$ => vertical deviation of B on the curve from the tangent at A



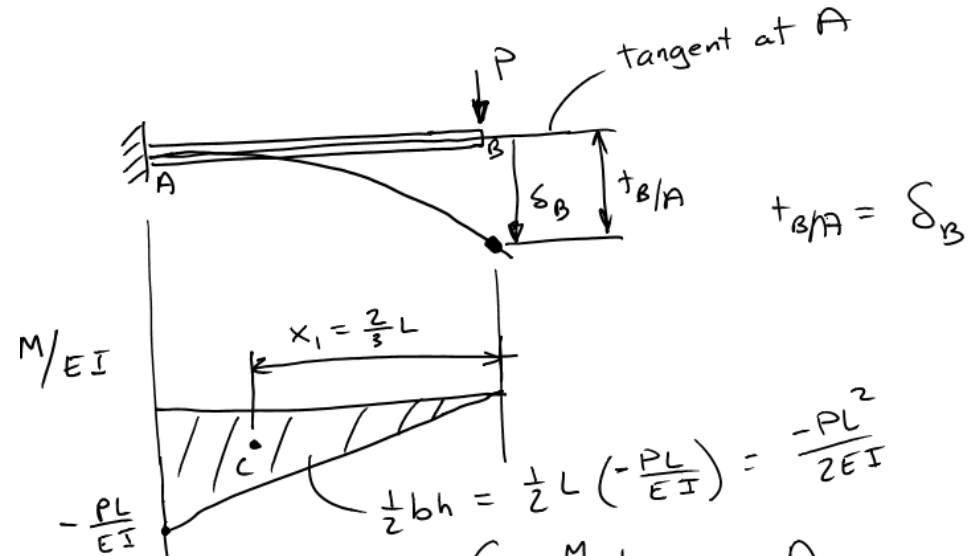
$$t_{B/A} = \int x_1 \frac{M}{EI}\,dx$$

$x_1 \Rightarrow$ Distance from the centroid of Area of the $\frac{M}{EI}$ diagram to the point on the curve you want find $t_{B/A}$ (B)



Appendix $\Rightarrow$ Areas & Centroids

- Cantilever Beam

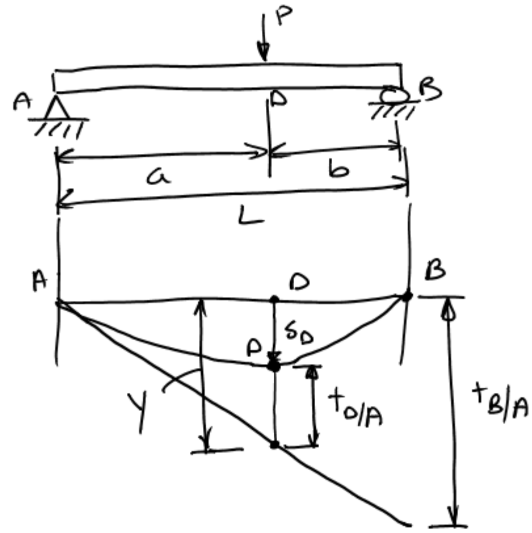


$$t_{B/A} = \int x_1 \frac{M}{EI}\,dx = x_1 A$$

$$\delta_B = -\underbrace{\left(\frac{2}{3}L\right)}_{x_1}\underbrace{\left(\frac{-PL^2}{2EI}\right)}_{A} = \boxed{\frac{PL^3}{3EI}}$$

δ is positive downwards

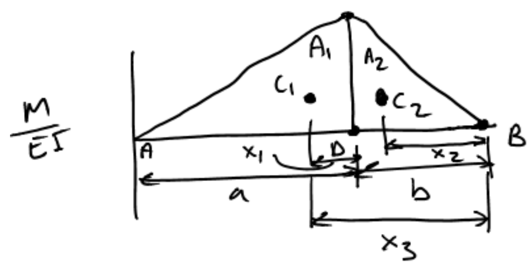
- Simply Supported Beam



Similar Triangles

$$\boxed{\frac{t_{B/A}}{L} = \frac{y}{a}}$$

$$\delta_D = y - t_{D/A}$$



$$t_{B/A} = x_3 A_1 + x_2 A_2$$

$$t_{D/A} = x_1 A_1$$

$$y = t_{B/A}\left(\frac{a}{L}\right)$$