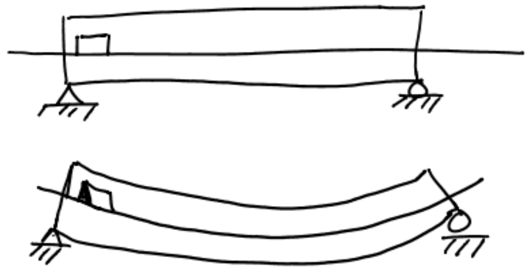
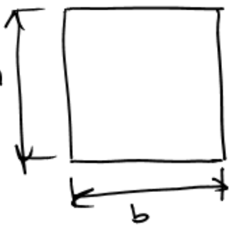
Shear Stresses in Beams of Rectangular Cross-Section





Shear Force

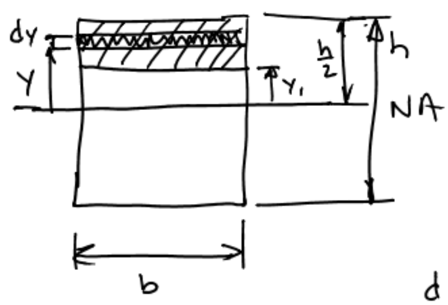
$$\boxed{\tau = \frac{VQ}{I\,b}}$$

base

Moment of Inertia about the Neutral Axis

$Q \Rightarrow$ First Moment of Area

$$Q = \int y\,dA$$



$$dA = b\,dy$$

$$Q = \int y\,dA$$

$$= \int_{y_1}^{h/2} y\,b\,dy$$

$$Q = b\left(\frac{1}{2}y^2\right)\Big]_{y_1}^{h/2}$$

$$= b\left(\frac{1}{2}\left(\frac{h}{2}\right)^2 - \frac{1}{2}y_1^2\right)$$

$$Q = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

Q is max when $y_1 = 0$

$$Q_{max} = \frac{bh^2}{8}$$

$$\tau_{max} = \frac{VQ_{max}}{Ib} = \frac{V\left(\frac{bh^2}{8}\right)}{\left(\frac{1}{12}bh^3\right)b}$$

$$\tau_{max} = \frac{3V}{2\underbrace{bh}_{A}}$$

$$\boxed{\tau_{max} = \frac{3V}{2A}}$$

Shear Stresses in Beams of Circular Cross-Section

- Evaluate stresses only at the neutral axis ($y_1 = 0$)
- Assume a constant stress distribution

$$I_{circle} = \frac{\pi}{4}r^4$$

$$Q = \int y\,dA = \bar{y}A$$

$$\bar{y} = \frac{4r}{3\pi}$$
$$A = \frac{1}{2}\pi r^2$$

$$Q = \left(\frac{4r}{3\pi}\right)\left(\frac{1}{2}\pi r^2\right) = \frac{2}{3}r^3$$

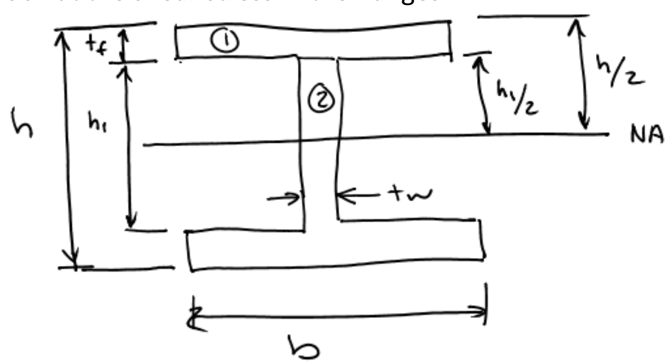
$$\tau_{max} = \frac{VQ}{Ib} = \frac{V\left(\frac{2}{3}r^3\right)}{\left(\frac{\pi}{4}r^4\right)\left(2r\right)}$$

$$= \frac{4V}{3\underbrace{\pi r^2}_{A}}$$

$$\boxed{\tau_{max} = \frac{4V}{3A}}$$

Shear Stresses in I-Beams

- Divide into rectangles
- Don't look at the shear stress in the flanges



$$A_1 = b t_f \qquad \bar{y}_1 = \frac{1}{2} t_f + \frac{h_1}{2}$$

$$A_2 = (t_w)\left(\frac{h_1}{2}\right) \qquad \bar{y}_2 = \frac{h_1}{4}$$

$$Q_{max} = \bar{y}_1 A_1 + \bar{y}_2 A_2$$

$$\tau_{max} = \frac{V Q_{max}}{I \underbrace{b}_{t_w}}$$