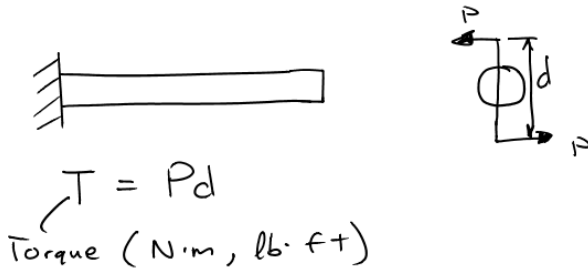
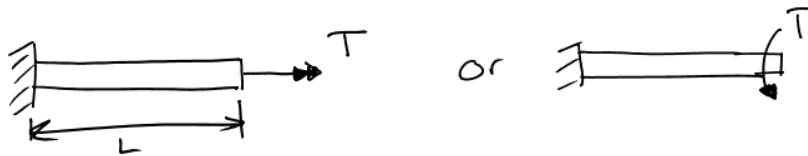


Torsion

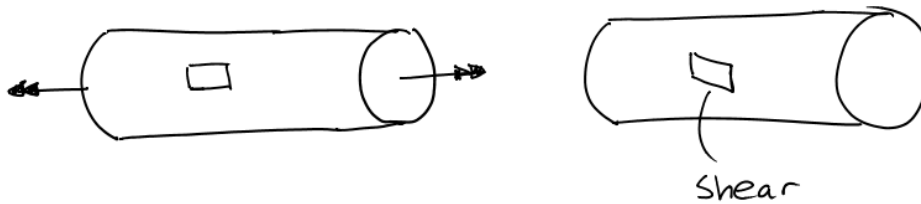
- Twisting of a bar due to moments of force called torques that produce a tendency of rotation about the longitudinal axis
- Torque is a couple moment



- Symbols



- Torsional Deformation

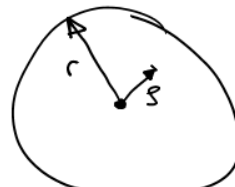


- Assumptions

- All cross-sections remain planar, circular, and straight
- Small deformations

- Terminology

- ϕ = Angle of Twist
- θ = Rate of Twist (per unit length)
- γ = Shear Strain
- τ = Shear Stress
- ρ = radius
- r = outer radius
- I_p = Polar Moment of Inertia



$$I_{p, \text{circle}} = \frac{\pi}{32} d^4 \quad I_{p, \text{tube}} = \frac{\pi}{32} (d_2^4 - d_1^4) \begin{matrix} \text{inner diameter} \\ \text{outer diameter} \end{matrix}$$

- Torsion Equations

$$\Theta = \frac{\phi}{L} \quad \gamma = \frac{\rho\theta}{L} \quad \gamma_{\max} = \frac{r\phi}{L}$$

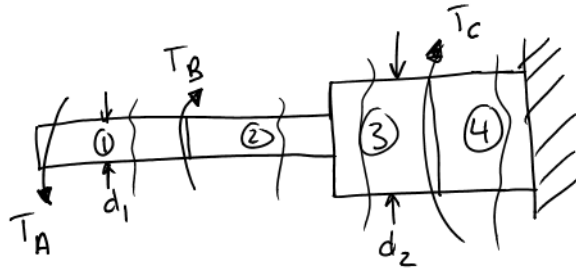
$$\tau = \frac{T\rho}{I_p} \quad \tau_{\max} = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{G I_p}$$

Also called J
Shear Modulus of Elasticity

Units for ϕ
 \Rightarrow radians \Rightarrow degrees

- Non-Uniform Torsion



4 segments

①

$$\sum T = 0 \quad +T_A + T_1 = 0$$

$$T_1 = -T_A$$

②

$$\sum T = 0 \quad T_2 + T_A - T_B = 0$$

$$T_2 = -T_A + T_B$$

③

$$T_3 = T_2 = -T_A + T_B$$

④

ENGR 2243 – Mechanics of Materials
Torsion

$$\rightarrow \sum T = 0 \Rightarrow T_4 + T_A - T_B - T_C = 0$$

$$T_4 = -T_A + T_B + T_C$$

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

$$\tau_{max} = \max |\tau_1, \tau_2, \tau_3, \tau_4|$$