

1.2

Fractions:

ex) $\frac{3}{4}$ ← numerator
 ← denominator

Natural numbers:

1, 2, 3, 4, 5, 6, ...

Mixed Numbers:

ex) $2\frac{1}{4}$

Improper fraction: has a numerator that is bigger than denominator

ex) $\frac{7}{2}$

Converting a mixed # to an improper fraction

Technically, a mixed # is a natural # plus a fraction.

ex) $2\frac{1}{4} = 2 + \frac{1}{4}$
 $= \frac{8}{4} + \frac{1}{4}$ ← rewrite 2 as $\frac{8}{4}$ to have common denominators.
 $= \boxed{\frac{9}{4}}$ ← improper fraction!

Shortcut,

$$A\frac{b}{c} = \frac{c \cdot A + b}{c}$$

ex) $2\frac{1}{4} = \frac{4 \cdot 2 + 1}{4}$
 $= \frac{8 + 1}{4}$
 $= \boxed{\frac{9}{4}}$

Converting Improper fractions to Mixed

Step 1: take numerator divided by denominator until you get a remainder (no decimals)

Step 2: mixed number is
 answer $\frac{\text{remainder}}{\text{original denominator}}$

ex) $\frac{7}{2}$

$$\begin{array}{r} 3 \\ 2 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

1 ← remainder!

Using Variables in fractions

In algebra we use letters to represent numbers. These letters are called variables

ex) $\frac{a}{b}$

Sometimes we choose values for these variables

ex) If we choose $a=4$ and $b=6$ then
 $\frac{a}{b} = \frac{4}{6}$

Factors, Prime & Composite #s

ex) $2 \cdot 3 = 6$ ← product - the answer when multiplying
 ↑ factors - the #'s being multiplied

We say 2 and 3 are factors of 6

Some #s have lots of factors!

ex) the factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

Since $1 \cdot 24 = 24$ $2 \cdot 12 = 24$ $3 \cdot 8 = 24$ $4 \cdot 6 = 24$

Some #s only have two factors
 (ex) the factors of 5 are 1 and 5
 • Since $1 \cdot 5 = 5$

Numbers such as 5 are called prime #s

prime number - a # whose only factors are 1 and itself (nothing else!)

composite number - a # that is NOT prime (aka it has more than two factors.)

(ex) 20 is composite because $4 \cdot 5 = 20$
 (and $2 \cdot 10 = 20$ and $1 \cdot 20 = 20$)

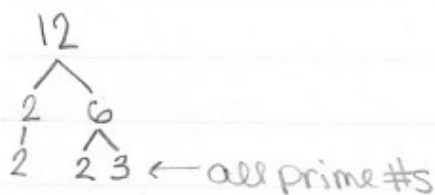
• 7 is prime because $1 \cdot 7 = 7$ but nothing else!

*Notice: when finding factors we only use the natural #s (that is no negatives, no decimals/fractions)

Prime factorization

Writing a composite # as a bunch of prime numbers multiplied together

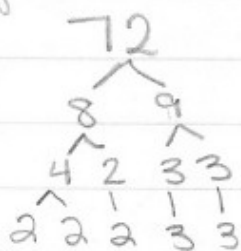
(ex)



So the prime factorization of 12 is $2 \cdot 2 \cdot 3$
 (notice: $2 \cdot 2 \cdot 3 = 4 \cdot 3 = 12 \checkmark$)

bigger the # the harder this is

(ex)



p.f. of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

Reducing fractions

Fundamental Principle of fractions: If a # is multiplied in the numerator & the denominator they "cancel"

in symbols: $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$

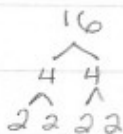
(ex) $\frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 4} = \frac{3}{4}$

To Completely Reduce a fraction

1. Find prime factorization of numerator
" denominator
2. Cancel whatever cancels.

(ex) $\frac{16}{24}$

$$\frac{16}{24} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3} = \frac{2}{3}$$



Note: this "cancelling" only works with **MULTIPLICATION!**

(non-ex) $\frac{2+3}{2+4}$ ← the 2's do NOT cancel!

Multiplying fractions

to multiply fractions

- multiply the numerators

- "

" denominators

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$\textcircled{\text{ex}} \quad \frac{2}{3} \cdot \frac{5}{6} = \frac{2 \cdot 5}{3 \cdot 6}$$

$$= \frac{10}{18} \quad \text{reduce} \rightarrow \frac{2 \cdot 5}{\cancel{2} \cdot 3 \cdot 3} = \frac{5}{9}$$

To divide fractions

- change the division to multiplication

- flip the 2nd fraction over (take the reciprocal)

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

$$\textcircled{\text{ex}} \quad \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \cdot \frac{4}{1}$$

$$= \frac{8}{3} \quad \text{reduce} \rightarrow \frac{2 \cdot 2 \cdot 2}{3 \cdot 3} \quad \text{nothing cancels} \rightarrow \frac{8}{3}$$

Some tricky things!

#1. all integers can be written as fractions w/ a 1 for a denominator $\textcircled{\text{ex}} \quad 5 = \frac{5}{1}$

#2. all mixed #'s can be written as improper fractions $\textcircled{\text{ex}} \quad 3\frac{2}{5} = \frac{17}{5}$

$$\textcircled{\text{ex}} \quad \frac{6}{7} \cdot 4\frac{1}{2} = \frac{6}{7} \cdot \frac{9}{2} = \frac{54}{14} \quad \text{reduce} \rightarrow \frac{27}{7}$$

$$\frac{4}{3} \div 2 = \frac{4}{3} \div \frac{2}{1}$$

$$= \frac{4}{3} \cdot \frac{1}{2} = \frac{4}{6} \quad \text{reduce} \rightarrow \frac{2}{3}$$

To add/subtract fractions

if the denominators are the same

- add/subtract numerators

- keep same denominator

if the denominators are different

- make denominators the same (get common den.)

- do above stuff

$$\textcircled{\text{ex}} \frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \boxed{\frac{4}{5}}$$

↑ same ↓

$$\frac{6}{9} - \frac{4}{9} = \frac{6-4}{9} = \boxed{\frac{2}{9}}$$

$$\textcircled{\text{ex}} \frac{1}{2} + \frac{3}{4}$$

↑ here the den. are different, so we want to make them the same (or get common denominators)

To get common denominators we want to "undo" some cancelling

$\textcircled{\text{ex}}$ Notice that $\frac{2}{4}$ when reduced is $\frac{1 \cdot 2}{2 \cdot 2} = \frac{1}{2}$
 So $\frac{2}{4} \rightarrow \frac{1}{2}$, now we want to "put the twos back in"

$$\frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} \rightarrow \frac{2}{4}$$

$$\textcircled{\text{ex}} \text{from above} \quad \frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$$

But how did I know to multiply the $\frac{1}{2}$ by $\frac{2}{2}$?

Two ways

1. Using Prime factorization
2. Using denominator shortcut

Prime factorization Method

- find prime factorization of both den.
- multiply each fraction by whichever factors it's missing. (on top & bottom)

$$\textcircled{\text{ex}} \frac{5}{18} + \frac{2}{21}$$

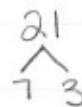
$$\frac{5}{2 \cdot 3 \cdot 3} + \frac{2}{7 \cdot 3}$$

$$\frac{5 \cdot 7}{2 \cdot 3 \cdot 3 \cdot 7} + \frac{2 \cdot 2 \cdot 3}{7 \cdot 3 \cdot 2 \cdot 3}$$

$$\frac{5 \cdot 7}{18 \cdot 7} + \frac{2 \cdot 2 \cdot 3}{21 \cdot 2 \cdot 3} = \frac{35}{126} + \frac{12}{126} = \boxed{\frac{47}{126}}$$



P.f. of 18 = 2 · 3 · 3



P.f. of 21 = 7 · 3

Denominator Shortcut

- multiply 1st fraction (on top & bottom) by den. of 2nd fraction
- multiply 2nd fraction (on top & bottom) by den. of 1st fraction.

$$\textcircled{\text{ex}} \frac{5}{18} + \frac{2}{21} = \frac{5 \cdot 21}{18 \cdot 21} + \frac{2 \cdot 18}{21 \cdot 18}$$

$$= \frac{105}{378} + \frac{36}{378} = \boxed{\frac{141}{378}} \quad \text{reduce} \rightarrow \frac{47 \cdot 3}{126 \cdot 3} = \boxed{\frac{47}{126}}$$