5.3

Mean

Variance

Expected Values

Using your calculator

5.3

Mean

Recall, $\bar{X} = \frac{\Sigma X}{n}$ $\mu = \frac{\Sigma X}{N}$ is 'mean.'

But what is the mean (or "average") you will get when rolling a die?
Could roll it an infinite # of times...
too long, need to formulas!

Mean of a Discrete Probability Distribution

$$\mu = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + \cdots + X_n \cdot P(X_n)$$
$$= \Sigma [X \cdot P(X)]$$

ex) Find the mean of rolling 1 die.

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

$$\mu = \Sigma [X \cdot P(X)] = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$$
$$= 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$$
$$= 21/6 = 3\,3/6 = 3.5$$

ex) Previously, we rolled two 4-sided die, X=sum. We got

X	2	3	4	5	6	7	8
P(X)	1/16	2/16	3/16	4/16	3/16	2/16	1/16

$$\mu = 2(1/16) + 3(2/16) + 4(3/16) + 5(4/16) + 6(3/16) + 7(2/16) + 8(1/16)$$
$$= \frac{2+6+12+20+18+14+8}{16}$$
$$= 80/16 = 5$$

ex

X	2	3	4	5
$P(X)$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

$$\mu = 2\left(\tfrac{2}{6}\right) + 3\left(\tfrac{1}{6}\right) + 4\left(\tfrac{2}{6}\right) + 5\left(\tfrac{1}{6}\right)$$
$$= \frac{4+3+8+5}{6}$$
$$= \frac{20}{6} = 3.\overline{3}$$

Procedure

multiply X by P(X) for all X

sum them all.

Variance

Recall, $\sigma^2 = \frac{\Sigma(x-\mu)^2}{N}$ and $\sigma = \sqrt{\sigma^2}$

For Discrete Probability Distribution

$$\sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$$

$$\sigma = \sqrt{\sigma^2}$$

Procedure

0. Find μ.

1. Square all values of X.
2. Multiply each X^2 by $P(X)$.
3. Sum all of these.
4. Subtract μ^2.

(5. take $\sqrt{\ }$ for st. dev.)

ex)

X	1	2	3	4
P(X)	1/7	2/7	3/7	1/7

0. $\mu = 1(1/7) + 2(2/7) + 3(3/7) + 4(1/7) = \frac{1+4+9+4}{7} = \frac{18}{7}$

1. $1^2 = 1$ $\quad 3^2 = 9$

   $2^2 = 4$ $\quad 4^2 = 16$

2. $1(1/7) = 1/7$ $\quad 9(3/7) = 27/7$

   $4(2/7) = 8/7$ $\quad 16(1/7) = 16/7$

3. $1/7 + 8/7 + 27/7 + 16/7 = 52/7$

4. $52/7 - (18/7)^2 = .816 = \sigma^2$

5. $\sigma = .9035$

or

$\mu = 1(1/7) + 2(2/7) + 3(3/7) + 4(1/7) = 18/7$

$\sigma^2 = 1^2(1/7) + 2^2(2/7) + 3^2(3/7) + 4^2(1/7) - (18/7)^2 = .816 \quad \sigma = \sqrt{.816}$

Expected Value

The expected value of a discrete random variable is the mean of the r.v., denoted $E(X)$.

$$\mu = E(X) = \Sigma X \cdot P(X)$$

ex Play a game. Chance of winning is $1/400$, if you win, you get \$125, cost \$1 to play

Let X = Gain

don't get \$1 back

a loss

X	\$124	-\$1
$P(X)$	$1/400$	$399/400$

$$E(X) = 124(1/400) + (-1)(399/400) = \frac{124}{400} - \frac{399}{400} = -.6875 \approx -\$.69$$

(Playing over a long period of time you would expect to "make" $-\$.69$)

Using Your Calculator

ex:

X	6	7	8	9	10
$P(X)$	.2	.2	.3	.2	.1

STAT

1: Edit

$L_1 = 6, 7, 8, 9, 10$

$L_2 = .2, .2, .3, .2, .1$

STAT

CALC

1: 1-Var Stats

2nd 1 (L_1)

,

2nd 2 (L_2)

ENTER

mean $= \bar{X} = 7.8$

$\sigma X =$ standard deviation ≈ 1.25

$\sigma^2 = (1.25)^2 = 1.56$