5.4

Binomial Distribution

Mean, Variance, Standard Deviation

5.4

Binomial Distribution

A <u>binomial experiment</u> is a probability experiment that satisfies the following four requirements

1. Each trial can have only two outcomes
   or outcomes that can be reduced to two outcomes
   These outcomes can be considered as either success or failure.
2. There must be a fixed number of trials.
3. The outcomes of each trial must be independent of each other.
4. The probability of a success must remain the same for each trial

A binomial experiment and its results give a special kind of discrete prob dist. called a <u>binomial distribution</u>.

Notation

P(S) = probability of success = p

P(F) = probability of failure = q

n = number of trials

X = the number of successes in n trials

Binomial Probability Formula

In a binomial experiment, the probability of exactly X successes in n trials is

$$P(X) = \frac{n!}{(n-X)!\,X!} \cdot p^{X} \cdot q^{n-X}$$

(ex) Toss 3 coins. Find P(2 Heads)

S = heads F = tail $n=3$

$P(S)=.5=p$ $P(F)=.5=q$

$$P(2)=\frac{3!}{(3-2)!\,2!}\cdot(.5)^2(.5)^{3-2}=\frac{6}{2}\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)=\frac{6}{16}=\frac{3}{8}$$

or

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

3 out of 8 or $P(2H)=3/8$.

(ex) I give you a 10 question multiple choice quiz. You haven't studied at all so you have to guess. Each question has 5 possible answers, one is right. Find $P(6)=?$

$p=1/5$ $n=10$

$q=4/5$

$$P(6)=\frac{10!}{(10-6)!\,6!}\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^{10-6}=.0055$$

(ex) Find $P(\text{pass})=P(6,7,8,9,10)$

$$=P(6)+P(7)+P(8)+P(9)+P(10)$$

$$=\frac{10!}{(10-6)!\,6!}\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^{10-6}+\frac{10!}{(10-7)!\,7!}\left(\frac{1}{5}\right)^7\left(\frac{4}{5}\right)^{10-7}+\frac{10!}{(10-8)!\,8!}\left(\frac{1}{5}\right)^8\left(\frac{4}{5}\right)^{10-8}+$$

$$\frac{10!}{(10-9)!\,9!}\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^{10-9}+\frac{10!}{(10-10)!\,10!}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^{10-10}$$

$2^1.0000001$

this would be tedious! So we use table B in the book, pg. 756 to approximate

Steps

1. find $n=10$
2. find $X=6,7,8,9,10$
3. find $p=.2$

$$\begin{array}{r} .0006 \\ +\ .001 \\ \hline \boxed{.007} \end{array}$$

Mean, Variance, Standard Deviation

For binomial distributions

$$\mu = n \cdot p$$
$$\sigma^2 = n \cdot p \cdot q$$
$$\sigma = \sqrt{n \cdot p \cdot q}$$

(ex) I have 25 students, 10 females, 15 males
I randomly select 3 people. P(Females) = ?
Find μ, σ^2, σ

$p = {}^{10}/_{25}$ $q = {}^{15}/_{25}$ $n = 3$

$\mu = 3({}^{10}/_{25}) = {}^{6}/_{5}$

$\sigma^2 = 3({}^{10}/_{25})({}^{15}/_{25}) = .72$

$\sigma = \sqrt{.72} \cong .849$