5.5

The Multinomial Distribution
The Poisson Distribution
The Hypergeometric Distribution
Summary of Discrete Distributions

5.5

The Multinomial Distribution

In order to use binomial distribution you need 2 outcomes (S,F). But what if there's more.

Multinomial Distributions

If X consists of Events $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_k$ which have corresponding probabilities $p_1, p_2, \ldots, p_k$ and X_1 is the # of times $\mathcal{E}_1$ will occur, ..., X_k is the # of times $\mathcal{E}_k$ will occur, then the probability that X will occur is

$$P(X) = \frac{n!}{X_1! \cdot X_2! \cdot \cdots \cdot X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdot \cdots \cdot p_k^{X_k}$$

where $X_1 + X_2 + \cdots + X_k = n$ and $p_1 + p_2 + \cdots + p_k = 1$

ex 3 Republicans, 2 Democrats, 1 undecided
Find P(1R and 1D and 1U)? Let $X = X_1, X_2, X_3$

$X_1 = 1 \quad X_2 = 1 \quad X_3 = 1 \qquad n = 3$

$p_1 = 3/6 \quad p_2 = 2/6 \quad p_3 = 1/6$

$$P(X) = \frac{3!}{1!1!1!}\left(\tfrac{3}{6}\right)^1\left(\tfrac{2}{6}\right)^1\left(\tfrac{1}{6}\right)^1 = 6\left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right)\left(\tfrac{1}{6}\right) = .1\overline{6}$$

The Poisson Distribution

If n is large and p is small, and occur over a period of time, given area or volume, etc. we use a Poisson Distribution.

The probability of X occurrences in an interval of time, volume, area, etc for a variable where λ (lambda) is the mean number of occurrences per unit (time, volume, area, etc.) is

$$P(X;\lambda) = \frac{e^{-\lambda}\lambda^{X}}{X!} \quad \text{where } X = 0, 1, 2, \ldots$$

ex) On average, I get 3 e-mails per day from my stats students.
Find the probability that, for any given day, I get at most 3 e-mails.

$$P(0;3)+P(1;3)+P(2;3)+P(3;3)$$

$$= \frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!}$$

or

Use table C, pg 762

$$.0498 + .1494 + .2240 + .2240 \cong .6472$$

The Hypergeometric Distribution

Given a population with only 2 types of objects (male, female; success, fail; etc.) such that there are a items of one kind and b items of another kind and a+b equals the total population, the probability of drawing X items of type A and n-X items of type B when selecting n items without replacement.

$$P(X) = \frac{{}_aC_x \cdot {}_bC_{n-x}}{{}_{a+b}C_n}$$

ex bucket with 4 red balls, 6 blue balls
Draw 3 balls. Find $P(2R, 1B)$?

$a=4 \quad n=3 \quad X=2 \quad a+b=10$

$b=6 \qquad n-X=1$

$$P(3R, 2B) = \frac{{}_4C_2 \cdot {}_6C_1}{{}_{10}C_3} = .3$$

1. 2 outcomes
2. a of one kind
   b of another kind
   a+b = total population
3. Select n items w/o replacement

Probability of drawing X items of type a and n-X of b. is $P(X) = \frac{{}_aC_x \cdot {}_bC_{n-x}}{{}_{a+b}C}$

Summary of Discrete Distributions

Binomial Distribution

1. Success, fail (independent)
2. P(S) stays constant
3. fixed number of trials

Multinomial Distribution

1. Two or more outcomes (independent)
2. $P(X_1), P(X_2), \ldots, P(X_R)$ stays constant
3. fixed number of trials

Poisson Distribution

1. n is large, p is small
2. occurs over a period of time, given area, volume, etc. (independent)

Hypergeometric Distribution

1. Success, fail
2. Without Replacement