6.3

The Standard Normal Distribution

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The Standard Normal Distribution

Since each normal distribution has it's own μ & σ we would need different info for each. This would be a pain, so we use the standard normal.

The <u>Standard Normal Distribution</u> is a normal distribution with $\mu=0$ and $\sigma=1$.

The formula for the standard normal distribution is $y=\frac{e^{-x^2/2}}{\sqrt{2\pi}}$

All normally distributed variables can be turned into standard normal variables by

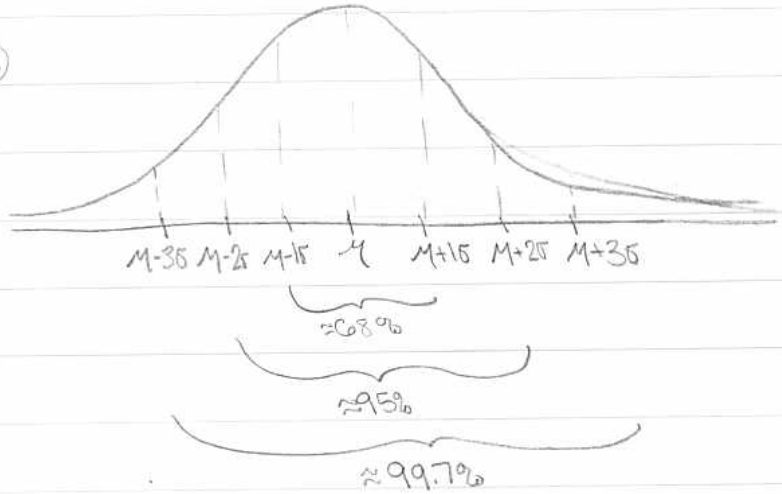
$$z=\frac{X-\mu}{\sigma}$$

Recall, this is called the z-value. the z-value is the number of σ X is from the mean.

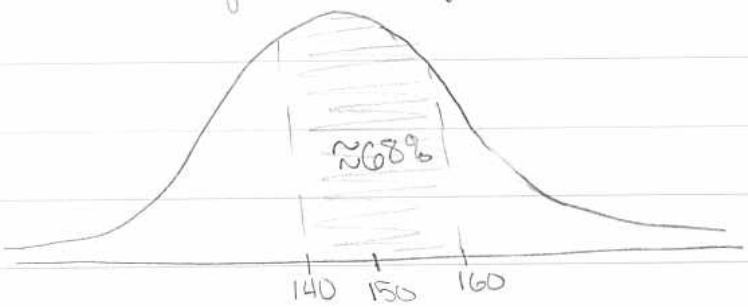
(ex) if $\mu=10$ and $\sigma=1$
then $X=11$ is 1 σ away from μ
$z=\frac{11-10}{1}=1$
$X=12$ is 2 σ away from μ
$z=\frac{12-10}{1}=2$
$X=8$ is 2 σ away from μ
$z=\frac{8-10}{1}=-2$ (below)

The important part of the normal curve is the area below it.

(ex)



For weights, $\mu = 150$, $\sigma = 10$



So ≈68% of people weigh between 140 and 160.
→ P(between 140 and 160) ≈ .68

To standardize this, shift the mean to 0

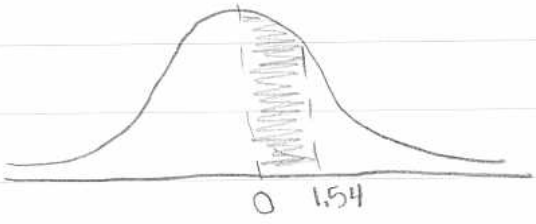
then squeeze that 68% into 2 lbs.

Finding Areas Under Standard Normal Curve

I knew the 68% since I was exactly 1 σ from μ, but what if I was 1.47 σ away

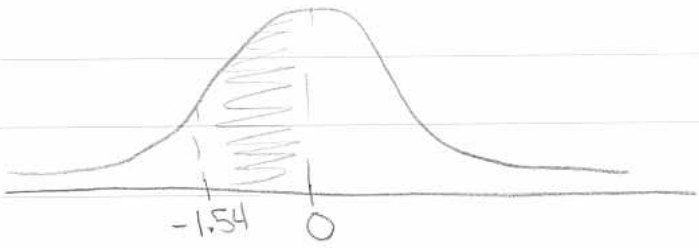
Table E on page 770 gives the ≈ area under the standard normal curve between the mean (0) and a value z (from 0 to 3.09)

(ex) Find the area under the standard normal curve between 0 and 1.54



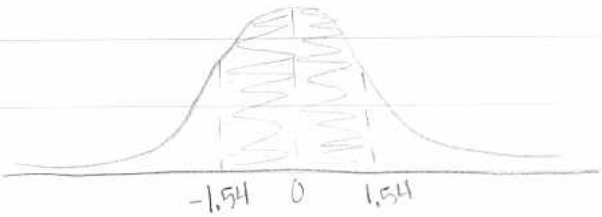
find z=1.5 in left column
move over to .04
so .4382

Find the area under the standard normal curve between 0 and -1.54



since normal is symmetric
.4382

Find the area under the standard normal curve between -1.54 and 1.54



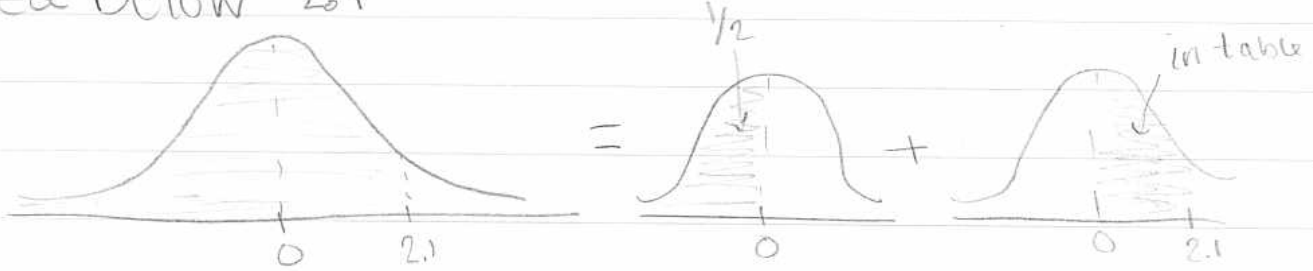
.4382 + .4382 = .8764

General Approach

1. Draw a picture
2. Shade the desired area
3. "Chunk" the shaded area into pieces you know
4. Add or subtract "chunks"

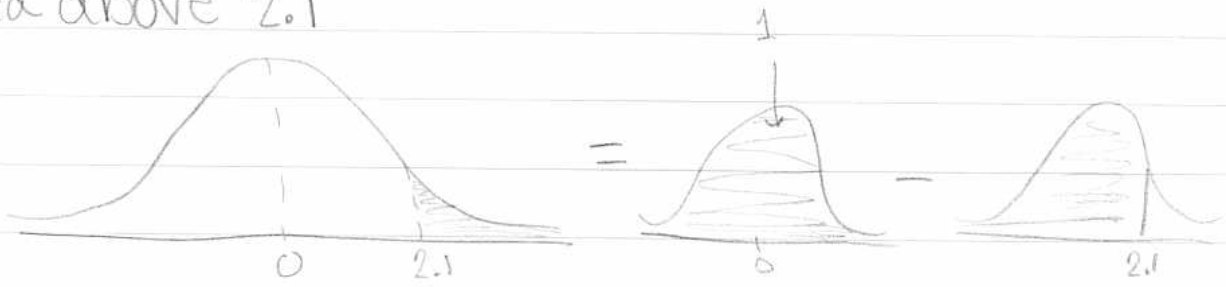
(*things you know: Area under whole curve is 1
Area under half curve is 1/2
Area under curve between 0 and z is in table (a positive z)
Curve is symmetric)

ex Area below 2.1



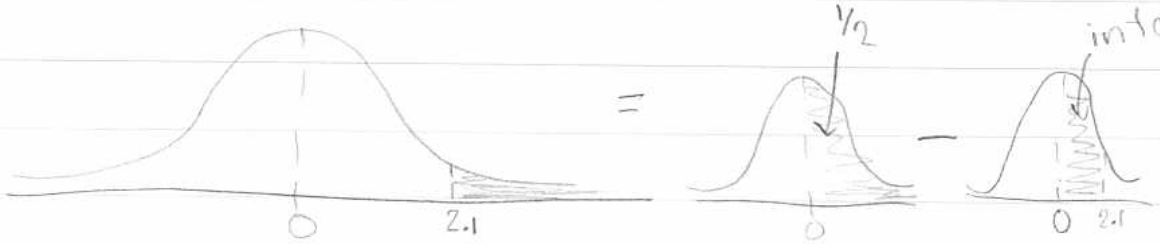
= .5 + .4821 = .9821

Area above 2.1



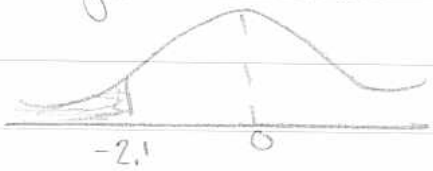
= 1 - .9821 = .0179

or



.5 - 4821 = .0179

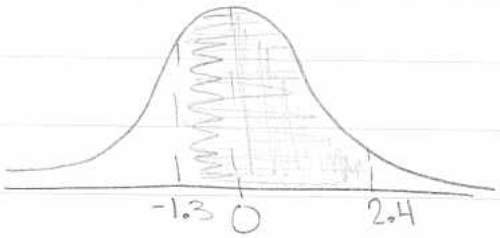
Same for below -2.1



Area above -1.72

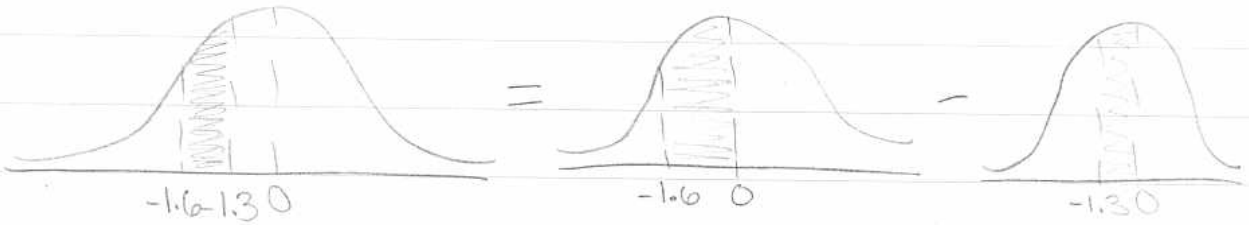
$= .5 + .4573 = \boxed{.9573}$

Area between -1.3 and 2.4



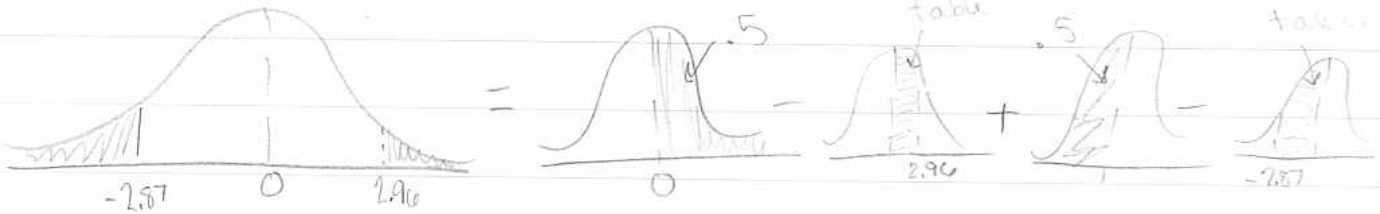
$.4032 + .4918 = \boxed{.895}$

Area between -1.3 and -1.6



$= .4452 - .4032 = \boxed{.042}$

Area above 2.96 and below -2.87



$.5 - .4985 + .5 - .4979 = .0036$

Important things to remember

- Area under whole curve is 1
- Area under half curve is .5
- Table gives area between 0 and z
- Curve is symmetric so area between 0 and z = area between 0 and -z.

Note- area is always positive

The Normal Curve as a Probability Distributio

The probability is the same as the area under the standard normal curve, for normally distributed data.

(ex) Find $P(0<z<1.67)$

$P(0<z<1.67)$ = area under st. nor. between 0 & 1.67

$P(0<z<1.67) = .4525$

$P(-1.3<z<2.4)$ = Area between -1.3 and 2.4

$= \boxed{.895}$

Sometimes need to work backwards

(ex) The area between 0 and z is .4265, what's z

$z = 1.45$

Might need to round if exact area is unavailabl