6.6

The Normal Approximation to the Binomial

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Recall, the Binomial Distribution has these characteristics
1. Fixed number of trials
2. Independent
3. Only 2 outcomes (reduced to two outcomes)
4. P(Success) stays constant.

The Binomial is determined by n & p.

When n is large and p is close to .5, the shape of the binomial distribution is similiar to the normal distribution
The larger n and the closer p is to .5, the better the approximation.

Rule of thumb: OK to use normal when
$n \cdot p \geq 5$ and $n \cdot q \geq 5$

ex $p = .4, n = 10 \rightarrow q = .6$
$n \cdot p = 10(.4) = 4 \not\geq 5$ so can't use it

$p = .4, q = .6, n = 100$
$(100)(.4) = 40 \geq 5, \ 100(.6) = 60 \geq 5$ so OK to use normal

Continuity Correction - Correction used when a continuous distribution is used to approximate a discrete distribution

ex In binomial distribution $P(X=5)$? In Normal $P(4.5 < X < 5.5)$.
" $P(X<5)$? " $P(X<4.5)$
" $P(X>5)$? " $P(X>5.5)$

$P(X \leq 5)$ $P(X < 5.5)$

$P(X \geq 5)$ $P(X > 4.5)$

See pg. 313

Recall, for Binomial $\mu = n \cdot p$, $\sigma = \sqrt{n \cdot p \cdot q}$
and $n \cdot p \geq 5$, $n q \geq 5$

Steps to use Normal to approximate Binomial

1. Can the normal be used ($np \geq 5$, $nq \geq 5$)
2. find μ & σ.
3. Write $P(X \_\_)$
4. Write using continuity correction. $P(X \_\_)$
5. find z-value(s)
6. find solution (table).

Ex 20% of Century College students say Math is favorite. If I randomly pick 400 students, what's the probability that exactly 75 students favor math.

1. $n = 400$, $p = .20$, $q = .80$
   $400(.2) = 80 \geq 5$ $400(.8) = 320 \geq 5$ OK
2. $\mu = n \cdot p = 400(.2) = 80$
   $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{(400)(.2)(.8)} = 8$
3. $P(X = 75)$
4. $P(74.5 < X < 75.5)$
5. $z_{74.5} = \frac{74.5 - 80}{8} = -.6875$ $z_{75.5} = \frac{75.5 - 80}{8} = -.5625$
6. $P(-.69 < Z < -.56) = .2549 - .2123 = .0426$

find P(more than 75 favor math) $= P(X > 75)$

4. $P(X > 75.5)$
5. $z_{75.5} = -.5625$
6. $P(Z > -.5625) = .2123 + .5 = .7123$