4.2

Basic Concepts

Classical Probability

Emperical Probability

4 Basic Rules

Law of Large Numbers

Subjective Probability

Basic Concepts

Probability started with gambling.

probability experiment - a chance process that leads to well-defined results called outcomes.

Outcome - the result of a single trial of a probability experiment.

ex probability experiment: flipping a coin rolling die. Outcome: Head or tails; 1,2,3,4,5,6

Sample space - the set of all possible outcomes of a probability experiment.

Experiment	Sample Space
Toss a coin	H, T
Roll a die	1,2,3,4,5,6
Toss 2 coins	HH, HT, TH, TT

ex find sample space for 2 die

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

tree diagram - a diagram to find sample spaces

ex Roll 3 die, what's the sample space?

Sample space is
HTH, HTT, HHH, HHT, TTH, TTT, THH, T

Tree diagram - works well for >2 die/coins/cards etc.

Three types of probability
1. Classical
2. Emperical (relative frequency)
3. Subjective

Event - consists of a set of outcomes

ex from 3 die
an outcome is HTH
an event is getting at least 2 H
{HTH, HHH, HHT, THH}

note: at least 2H
2H
at most 2H

4 Basic Rules

1. $0 \leq P(E) \leq 1$
2. If E cannot occur, then $P(E)=0$
3. If E has to occur, then $P(E)=1$
4. The sum of the P( ) of all outcomes in S is 1.

ex rolling die

$0 < P(1) = \ldots = P(6) = 1/6 < 1$

$P(7)=0$

$P(1 \text{ or } 2, \ldots, \text{or } 6)=1$

$P(1)+P(2)+\cdots+P(6) = \frac{1}{6}+\cdots+\frac{1}{6} = \frac{6}{6} = 1$

■

<u>Complement Event</u> - the complement event of E, is the event containing all outcomes not in E, denoted $\bar{E}$.

ex Let E = "getting a 1,2,3,4,5" when rolling a die then $\bar{E}$ = "getting a 6". ■

ex Selecting a student in class E = "getting a boy"

$\bar{E}$ = "getting a girl".

If $\hat{E}$ = "getting a 20-yr old"

$\bar{E}$ = "getting someone whose not 20" ■

Classical Probability

Classical Probability assumes that all outcomes are equally likely to occur.

ex rolling a die (note: always fair, unless stated)

sample space is 1,2,3,4,5,6

Probability of $1 = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

We call these events equally likely events.

The probability of any event E is

$$\frac{\#\text{ of outcomes in } E}{\#\text{ of outcomes in Sample Space}}$$

denoted

$$P(E) = \frac{n(E)}{n(S)} \qquad S = \text{sample space}$$

P(E) is expressed as a fraction, decimal, percentage

ex from tree-diagram example, find P(E) when E = "at least 2 H"

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2} = .5$$

ex 52-card deck, find

$P(7) = 4/52 = 2/26$

$P(7 \text{ or } 2 \text{ of hearts}) = ? = 5/52$

4 possible 7's

1 possible 2 of hearts

5 possible in E

Rules for Complementary Events

$P(E) = 1 - P(\bar{E})$ or $P(\bar{E}) = 1 - P(E)$ or $P(E) + P(\bar{E}) = 1$

*note: typo in book

Sometimes $P(\bar{E})$ is easier to find than $P(E)$, in these cases we can use $P(E) = 1 - P(\bar{E})$ to find $P(E)$.

ex $E =$ "picking a student whose not 20"

I would need to ask whose 1, 2, 3, …, 19, 21, 22, …

$\bar{E}$ is easier to find

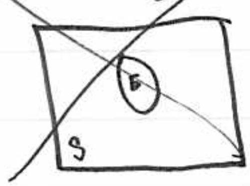
I would ask whose 20? Then find $P(\bar{E})$, then use Rules for E & $\bar{E}$.

Say 5 students are 20 (25 students in class)

$P(\bar{E}) = 5/25 = 1/5$

so $P(E) = 1 - 1/5 = 4/5$.

~~Venn Diagram~~



Emperical Probability

<u>Emperical Probability</u> - rely on actual experience to determine probability.

ex) Instead of assuming $P(H)=P(T)$, you flip a coin a few thousand times and find your ~~P~~ $P(H)$ and $P(T)$ from the experiment.

Emperical Probability Formula

$$P(E) = \frac{\text{frequency for the class } E}{\text{total frequencies in all classes}} = \frac{f}{n}$$

notice the wording, the data is often summarized in a frequency distribution.

ex) I flip a coin 10,000 times and get

class	freq
H	5,049
T	4,951

$P(H) = \frac{5{,}049}{10{,}000}$ $\qquad$ $P(T) = \frac{4{,}951}{10{,}000}$

ex) 25 people in class, 10 got A's, 8 got B's, 3 got C's, 1 got D, 3 got F

Class	freq
A	10
B	8
C	3
D	1
F	3

$P(A) = \frac{10}{25} =$

$P(B) = \frac{8}{25}$

$P(C) = \frac{3}{25}$

$P(D) = \frac{1}{25}$

$P(F) = \frac{3}{25}$

ex cont'd

Class	freq.	Cumm. freq.
A	10	10
B	8	18
C	3	21
D	1	22
F	3	25

$P(\text{C or better}) = \frac{21}{25} = P(\text{At least a C})$

$P(\text{D or F}) = 1 - \frac{21}{25} = \frac{4}{25}$

$P(\text{A or C}) = \frac{13}{25}$

Law of Large Numbers

The Law of Large Numbers says that the empirical probability will approach the Classical probability as the number of trials increases.

(ex) The empirical probability of ~~rollin~~ getting a H on a coin flip should approach the classical probability of getting a H, that is $\frac{1}{2}$.

So if I flip a coin 4 times, my Empirical Probability may be far from $\frac{1}{2}$ but if I flip a coin 4,000 times, the Empirical Probability will be closer to $\frac{1}{2}$.

Subjective Probability

Subjective Probability uses educated guesses or estimates.

ex) What's the probability of dieing of cancer in 1739?
We don't know how many people had cancer in 1739, but we could look through Dr's notes and try to "guesstimate" how many there were.