4.3

The Addition Rules for Probability

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Mutually exclusive - events which cannot occur at the same time

ex) You draw one card from a full deck.
The event

a. Getting a 4 and a red
↑ not mutually exclusive.

b. Getting a diamond and a black
↑ mutually exclusive.

c. Getting an odd # (J-o, K-o) and less than 5.
↑ not mutually exclusive (could get a 3.)

d. Getting a face card and less than 5.
↑ mutually exclusive

When two events A and B are mutually exclusive, the probability that A or B will occur is

$$P(A \text{ or } B) = P(A) + P(B)$$

ex) A bin contains 4 red marbles, 5 blue marbles, and 3 yellow marbles. You pick one marble at random. Find $P(R \text{ or } Y)$?

R and Y are mutually exclusive, so

$$P(R \text{ or } Y) = P(R) + P(Y)$$
$$= 4/12 + 3/12 = 7/12$$

If A and B are NOT mutually exclusive then

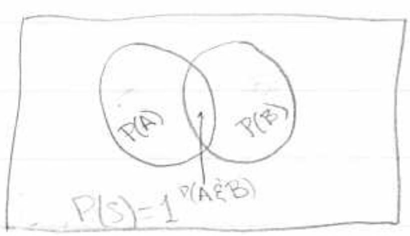
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

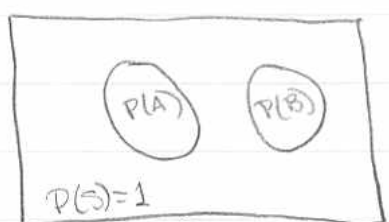
ex You roll a die, let A = rolling an even number, B = rolling a number less than 3.

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= P(2,4,6) + P(1,2) - P(2) \\ &= \tfrac{3}{6} + \tfrac{2}{6} - \tfrac{1}{6} \\ &= \tfrac{4}{6} = \tfrac{2}{3}. \end{aligned}$$

Venn Diagrams

Some people like to use Venn Diagrams to help visualize these rules.

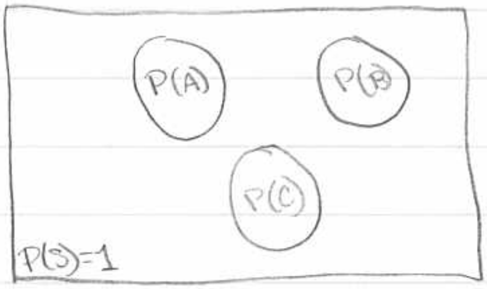




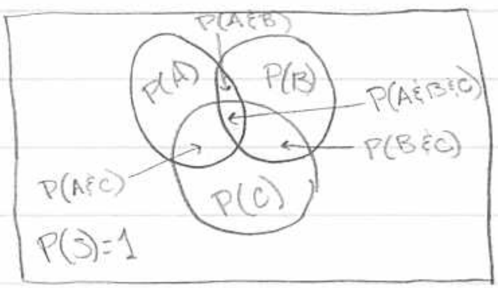
Since P(A&B) is counted twice we need to subtract out one P(A&B)

$$P(A \text{ or } B) = P(A) + P(B)$$

Venn Diagrams help us extend the idea to 3 events



So,
if A,B,C are mutually exclusive
then $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$.



So
if A,B,C are not mutually exclusive then
$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \& B) - P(B \& C) - P(A \& C) + P(A \& B \& C)$