4.4

Independent & Dependent Events
Multiplication Rules
Conditional Probability
Probabilities for "at least"

Independent & Dependent Events

Independent Events - events A & B are independent if A occurring does not affect the probability of B occurring.

ex) Roll a die twice A = get a 3 on first roll
B = get an even on second roll.

Draw a card and toss a coin.
A = drawing a Ace
B = getting a T

Dependent Events - when the probability of B changes or depends on A.

ex) A = registering for a prize
B = winning the prize
The probability that you win depends on whether you registered

ex) Draw 2 cards w/o replacement
A = J, Q, K on first draw
B = A on second draw

Conditional probability - the probability of an event, B, happening given A happened.
denoted $P(B|A)$.

(ex) Let $A = J, Q, K$ on first draw and
$B = A$ on 2^{nd} draw from above ex.
(that is, 2 draws w/o replacement)
$P(B|A) = 4/51$

note: if A and B are independent then $P(B|A) = P(B)$

(ex) Let $A = J, Q, K$ on 1^{st} draw, $B = A$ on 2^{nd} draw
where you draw 2 cards WITH replacement
$P(B|A) = 4/52 = P(B)$

Multiplication Rules

If A & B are independent
$P(A \text{ and } B) = P(A) \cdot P(B)$
If A & B are dependent
$P(A \text{ and } B) = P(A) \cdot P(B|A)$

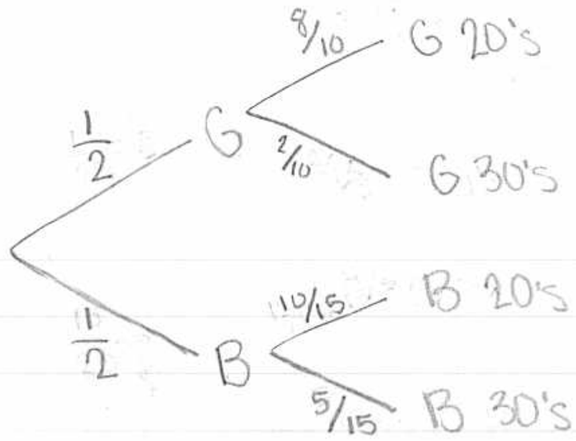
ex) from card example.
With replacement
$P(A \text{ and } B) = P(J,Q,K \text{ on } 1^{st} \text{ AND } A \text{ on } 2^{nd}) =$
$= P(J,Q,K \text{ on } 1^{st}) \cdot P(A \text{ on } 2^{nd})$
$= (12/52)(4/52)$
$= .01775$

Without replacement
$P(A \text{ and } B) = P(J,Q,K \text{ on } 1^{st}) \cdot P(A \text{ on } 2^{nd} | J,Q,K \text{ on } 1^{st})$
$= (12/52)(4/51)$
$= .018$

Do the example w/ students →

Tree Diagrams can be helpful picturing conditional probability

ex) 25 students in class, 10 girls, 15 boys
8 girls are in their 20's, 2 in 30's
10 boys are in their 20's, 5 in 30's
I flip a coin, H → pick Boy, T → pick a Girl.
Find P(pick a 20's)?



$P(20) = P(G \text{ and } 20) + P(B \text{ and } 20)$

Are these independent or dependent?

dependent. $P(20) = \begin{cases} 8/10 & \text{if } G \\ 10/15 & \text{if } B \end{cases}$

So

$$P(20) = P(G)P(20|G) + P(B)P(20|B)$$
$$= (1/2)(8/10) + (1/2)(10/15)$$
$$= 8/20 + 10/30 = 24/60 + 20/60 = 44/60 = .7\overline{3}$$

$$P(30) = P(G)P(30|G) + P(B)P(30|B)$$
$$= (1/2)(2/10) + (1/2)(5/15)$$
$$= 2/20 + 5/30 = 6/60 + 10/60 = 16/60 = .2\overline{6}$$

or

$$P(30) = 1 - P(\text{not } 30) = 1 - P(20)$$ ← easier?
$$= 1 - 44/60 = 16/60 = .2\overline{6}$$

Problems

1 girl in 20's

2 boys 1 in 20's, 1 in 30's

Flip a coin, H → Boys

What's the chance that 20 yr old is picked .50

T → Girls

What's the chance that 20 yr old is picked 1.0

So the 20 yr old girl has a better chance of being picked.

Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

ex) P(A on first exam) = .42
P(A on first exam AND A in Class) = .34
find P(A in Class | A on first exam)?

$$P(\text{A in Class} | \text{A on } 1^{st} \text{ exam}) = \frac{P(1^{st} \text{ AND class})}{P(1^{st})}$$

$$= \frac{.34}{.42} = .81$$

ex) I ask college students and Professor's
Is tuition too high?

	Students	Profs	total
Too High	18	7	25
OK	5	6	11
Too Low	2	2	4
Total	25	15	40

Find
P(Said OK, given Prof) = P(OK | Prof)

$$= \frac{P(OK \ \& \ Prof)}{P(Prof)}$$

$$= \frac{6/40}{15/40} = 6/15 = .4$$

P(Are a Prof | Said OK) = P(Prof | OK)

$$= \frac{P(Prof \ \& \ OK)}{P(OK)}$$

$$= \frac{6/40}{11/40} = 6/11 = .\overline{54}$$