Probabilities for "At least"

ex) Play a game, draw 4 cards with replacement. You win

$1 - 1A

$2 - 2A

$3 - 3A

$4 - 4A

Find P(win$)?

$P(\text{win}\$) = P(1A) + P(2A) + P(3A) + P(4A)$

$P(1A) = P(A_1) + P(A_2) + P(A_3) + P(A_4)$

$P(2A) = P(A_1 \& A_2) + P(A_1 \& A_3) + P(A_1 \& A_4) + P(A_2 \& A_3) + \cdots$

This would take a while.

Easier find $P(\text{loose}) = P(0\ \text{Aces})$

then $P(\text{win}) = 1 - P(\text{loose})$

$P(\text{loose}) = P(0\ \text{Aces}) = P(\text{no } A_1 \text{ and no } A_2 \text{ and no } A_3 \text{ and no } A_4)$

Are they independent?

Yes, "with replacement"

So use $P(A \& B) = P(A)P(B)$ extended

$P(0\ \text{Aces}) = P(\text{no } A_1) \cdot P(\text{no } A_2) \cdot P(\text{no } A_3) \cdot P(\text{no } A_4)$

$= (48/52)(48/52)(48/52)(48/52)$

$= \frac{20736}{28561}$

$P(\text{loose}) = .726$

$P(\text{win}) = 1 - .726 \approx .274$

Can do this for "at most" too