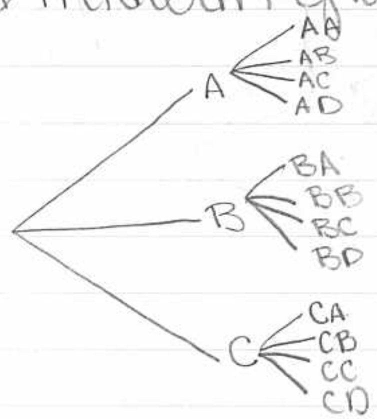
4.5

The Fundamental Counting Rule
Permutations
Combinations

Fundamental Counting Rule

In a sequence of n events, where the first event has R_1 possible outcomes, the second has R_2 possible, etc.
The number of possible sequences is
$R_1 \cdot R_2 \cdot R_3 \cdot \cdots R_n$

ex) Midterm grades and final grades (M: A-C, F: A-D)



Midterm = $R_m = 3$
Final = $R_f = 4$
so $\underline{3} \cdot \underline{4} = 12$

ex) Our ID #s are 8 digits long. How many ID #s before they have to repeat?
$\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 100,000,000$

What if I don't want repeating numbers. (ie only one of each digit.)
$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 1,814,400$

ex) How many "words" can be written using the letters JASON (use each letter once)
$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 120$
How many five letter "words" are there
$\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} = 11,881,376$
$\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{23} \cdot \underline{22} = 7,893,600$ ← no repeats

Think of the possible outcomes as a "word"

ex Toss 6 coins, how many possible outcomes

HHHHHH, or HHHHHT, etc.

↑ "words" ↑

$\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} = 64$

ex Draw 6 cards from a deck w/o replacement

234567, AJ257Q, etc.

$\underline{52} \cdot \underline{51} \cdot \underline{50} \cdot \underline{49} \cdot \underline{48} \cdot \underline{47} \cong 14658134400$

Factorial Notation, $n! = n(n-1)(n-2)\cdots(2)(1)$ and $0! = 1$

read "n factorial"

ex $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3628800$

Make sure you know button on calculator

\# $\boxed{\text{math}} \to \boxed{\text{PROB}} \to \boxed{4: !}$

ex 6 question multiple choice (A,B,C,D)

$\underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4}$

6 question matching (A,B,C,D,E,F)

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

Permutations

permutations - an arrangement of objects in a specific order, denoted ${}_nP_r$

Permutations - Order Matters.

ex) I decide to rank my 7 favorite foods, best to worst. How many ways can I order them.

$\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 7! = 5040$

In this example, all of my choices were used up (I had 7 possible dinners and I ranked 7 dinners.)

ex) I tell my husband my 7 favorite foods and he ranks is favorite 3.

$\underline{7} \cdot \underline{6} \cdot \underline{5} = 210$

In general,

n objects total, use r objects at a time, when order matters

$${}_nP_r = \frac{n!}{(n-r)!}$$

ex) from above

$n=7,\ r=3$

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

note $\frac{7\cdot 6\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 7\cdot 6\cdot 5$

Combinations

Combination - selection of distinct objects with no regard to order; denoted ${}_nC_r$

Combination - Order Does NOT matter

ex) I have 4 students, A, B, C, D, how many ways can I pair them up?

AB
AC BC
AD BD CD

← Combination

If I assign a 'team leader'

AB BA
AC CA BC CB
AD DA BD DB CD DC

← Permutation

In general, n total objects, using r objects the number of combinations is

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

ex) from above,

$${}_4C_2 = \frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1} = \frac{12}{2} = 6.$$

ex) There are 48 problems for 4-5, your next assignment should be 14 questions, how many ways can I choose the assignment.

$${}_{48}C_{14} = \frac{48!}{(48-14)!\,14!} \cong 4.8\times10^{11}$$

ex) there are 5 A students, 4 B students, 2 C students
I want a group of students
so that there are
2 A's, 2 B's, 1 C
how many ways can I choose them?

$_5C_2 = 10$
$_4C_2 = 6$
$_2C_1 = 2$

Think of this as three events (A=choosing A, ...)
By Fundamental Counting Rule
$\underset{A}{\underline{10}} \cdot \underset{B}{\underline{6}} \cdot \underset{C}{\underline{2}} = 120$ ways

Know $_nP_r$ and $_nC_r$ buttons on calculators as a check, also need to be able to manipulate formulas.

n $\boxed{MATH}$ $\boxed{PRB}$ $\boxed{2: nPr}$ / $\boxed{3: nCr}$ r

See table 4.1