7.2

Confidence Intervals (σ known or n ≥ 30)

Formula for C.I. of Mean

Sample Size

Using Your Calculator

Confidence Intervals (σ known or $n \geq 30$)

Sometimes we can't find the exact value for a parameter (ex. mean age of Americans) so we find an estimate instead. (ex. mean, $\bar{x}$, age of a sample of Americans.) This kind of estimate is called a point estimate

There is no way of knowing how close a point estimate is to the parameter. So we use interval estimates.

Interval Estimate - an interval or range of values used to estimate a parameter

ex mean age of Century Student
Point estimate is 23.6 yrs.
Interval estimate is 22 - 25.2

note: the actual μ might or might not be in 22-25.2

But how sure am I that μ is w/in 22-25.2 yrs?
what about w/in 15-45 yrs?
what about w/in 10-12 yrs?

I'm "almost 100% sure" it's w/in 15-45 yrs and "almost 0% sure" it's w/in 10-12 yrs.

This is the idea of a confidence level.

Confidence level - the probability that the interval estimate will contain the parameter.

(ex) 99.9% Confident Mage of century is b/in 15-45.
↑ Confidence level

What if I know I want to be 99.9% sure that μ is b/in an interval, then I choose the interval according to that 99.9%.

Confidence interval - a specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate

(ex) I want to be 99.9% sure that μ is in interval so the interval is 15-45.
↑ confidence interval.

Most common confidence intervals used are 90%, 95%, 99%.

Formula for the Confidence Interval of the mean for A specific α.

$$\bar{X} - Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

where

$Z_{\alpha/2}$ is the z value from the normal w/ $\alpha/2$ area under the curve in each tail.

α = 1-confidence level

(ex) Century Students Age. I sample 100 students

$\bar{X} = 23.6$ yrs. $\sigma = 1.2$ yrs.

Find 95% confidence interval of μ.

What do I know? $n=100$, $\bar{X}=23.6$, $\sigma=1.2$, $\alpha=.05$

$$23.6 - 1.96\left(\frac{1.2}{\sqrt{100}}\right) < \mu < 23.6 + 1.96\left(\frac{1.2}{\sqrt{100}}\right) \quad (.2352)$$

$$\boxed{23.3648 < \mu < 23.8352}$$

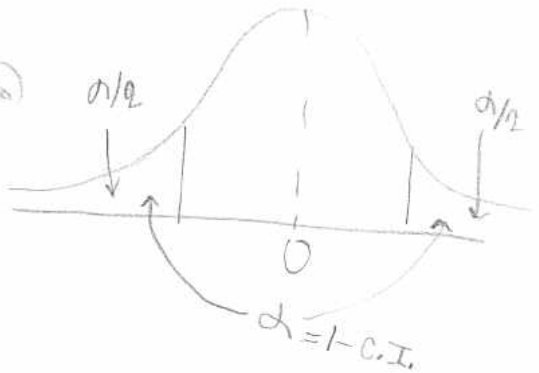
$\alpha/2 = .025$

$.5 - \alpha/2 = .475$

$Z_{\alpha/2} = 1.96$

So I'm 95% Confident that μ is between 23.3648 & 23.8352

They do α funny!



maximum error of an estimate - the maximum amount that you are probably off.

maximum error is $Z_{\alpha/2}(\sigma/\sqrt{n})$

When $n \geq 30$, s can be used for σ.

Sample Size

How big of a sample do I need?

Formula for Minimum Sample Size

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$ round up if needed

where E is the maximum error of estimate.

(ex) I want to be 90% confident that I estimate Century students age w/in 1 years (assume $\sigma = 2.5$)

$\alpha = .1$ $(1-.90)$, $\alpha/2 = .05$, $z_{\alpha/2} = 1.65$, $E = 1$

$$n = \left(\frac{1.65(2.5)}{1}\right)^2 = \lceil 17.01 \rceil = 18$$

Using Your Calculator

Given Data

1. Enter data into L_1
2. STAT → TESTS
3. Z-Interval
4. Select Data
5. Enter data
6. Calculate → Enter

Given Stats

1. STAT → TESTS
2. Z-Interval
3. Select Stats
4. Enter data
5. Calculate → Enter