

7.5

C.I. for Variances and Standard Deviat

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So far, we've found C.I. for mean and proportions.

To find C.I. for σ^2 and σ , we use the chi-squared distribution, χ^2 .

χ^2 is dependent on d.f. = $n-1$.

To use table

1. find $\alpha = 1 - \text{C.I.}$
2. find $\alpha/2$
3. Look $\alpha/2$, d.f. up in table, this is χ^2_{right}
4. find $1 - \alpha/2$
5. Look $1 - \alpha/2$, d.f. up in table, this is χ^2_{left}

Formula for the C.I. for Variance

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}} \quad \text{d.f.} = n-1$$

Formula for C.I. for Standard Deviation

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\text{right}}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{\text{left}}}} \quad \text{d.f.} = n-1$$

ex) Find 90% C.I. for the σ^2 and σ of an exam if a sample of 30 students has $s=2$

Find χ^2_{right} and χ^2_{left}

$$1. \alpha = 1 - .90 = .10$$

$$2. \alpha/2 = .05$$

$$3. \chi^2_{\text{right}} = 42.557$$

$$4. 1 - .05 = .95$$

$$5. \chi^2_{\text{left}} = 17.708$$

C.I. for σ^2 :

$$\frac{29.4}{42.557} < \sigma^2 < \frac{29.4}{17.708}$$

$$2.725 < \sigma^2 < 6.55$$

C.I. for σ :

$$\sqrt{2.725} < \sigma < \sqrt{6.55}$$

$$1.65 < \sigma < 2.56$$