

## Math 1400 Practice Exam

1. The point  $P(2, 1)$  lies on the curve  $y = \sqrt{x-1}$ . If  $Q$  is the point  $(x, \sqrt{x-1})$ , use your calculator to find the slope of the secant line  $PQ$  (correct to six decimal places) for the following values of  $x$ :

1.9, 1.99, 1.999, 2.5, 2.1, 2.01, 2.001

Guess the value of the slope of the tangent line to the curve at  $P(2, 1)$ .

2. Let  $f(x) = \frac{x^2 - 2x}{x^2 - x - 2}$ . Guess the value of  $\lim_{x \rightarrow 2} f(x)$  by evaluating the function at the given numbers. Then use the limit laws to show that you are correct.

1.9, 1.99, 1.999, 2.5, 2.1, 2.01, 2.001

3. Find the following limits

a.  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

b.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right)$

c.  $\lim_{x \rightarrow \infty} \frac{3x+5}{x-4}$

d.  $\lim_{x \rightarrow \infty} \frac{3x^2+5}{x-4}$

e.  $\lim_{x \rightarrow -\infty} \frac{3x+5}{x^4-4}$

f.  $\lim_{x \rightarrow 4^-} \frac{3x+5}{x-4}$

g.  $\lim_{x \rightarrow 4^+} \frac{3x+5}{x-4}$

h.  $\lim_{x \rightarrow 4} \frac{3x+5}{x-4}$

i.  $\lim_{x \rightarrow 2} \frac{3x+5}{x-4}$

4. Prove that  $\lim_{x \rightarrow 0} (\sqrt{x^3 - x^2}) \sin\left(\frac{8}{x}\right) = 0$

5. a. Show that  $\ln x = e^{-x}$  has a root on the interval  $(1, 2)$ .

b. Use the Intermediate Value Theorem to show that there is a number between  $-2$  and  $1$  with the following property:

The number is exactly 1 more than its cube.

6. Is  $f(x)$  continuous? Explain/show some work.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x-3 & \text{if } x > 1 \end{cases}$$

If  $f(x)$  is discontinuous, what linear equation could you replace “ $2x-3$ ” with to make  $f(x)$  continuous?

7. Sketch a graph with a jump discontinuity at  $x = 2$ , an infinite discontinuity at  $x = -3$ , that is continuous everywhere else.

8. Show that  $f(x) = x^2 + \sqrt{7-x}$  is continuous at  $a = 4$

9. Do #5 on page 106 of the text.

10. Let  $f(x) = x^2 - 3x$ .

a. Find an equation of the tangent line at the point  $(1, -2)$

b. Use the definition of a derivative to find  $f'(x)$ .

11. The position (in meters) above the ground of a baseball moving through the air is given by  $s(t) = -t^2 + 6t + 1$ , where  $t$  is measured in seconds.

a. Find the average velocity over each time interval:

$$[3, 4]$$

$$[3.5, 4]$$

$$[4, 5]$$

$$[4, 4.5]$$

b. What is the meaning of  $s'(t)$ ? What are its units?

c. What does  $s(5) = 6$  mean?

d. What does  $s'(5) = -4$  mean?

e. Find the instantaneous velocity when  $t = 4$ .

f. When will the baseball hit the ground?

g. With what velocity will the baseball hit the ground?

h. Find  $s'(6)$ ? Use this to find an equation of the tangent line at  $(6, 1)$

i. Find  $s'(a)$  using the definition of the derivative.

12. Do #5 on page 165

13. Do # 31-34 on page 167

14. Do #11 on page 173

15. Sketch the graph of a function such that

$$f'(1) = f'(-1) = 0,$$

$$f'(x) < 0 \text{ if } |x| < 1,$$

$$f'(x) > 0 \text{ if } 1 < |x| < 2,$$

$$f'(x) = -1 \text{ if } |x| > 2,$$

$$f''(x) < 0 \text{ if } -2 < x < 0,$$

inflection point  $(0, 1)$

Anything covered in class, on the homework, or in the text.