## 2.2 The Limit of a Function

We have seen that limits arise when finding the slope of a tangent line to a curve, or when finding the instantaneous velocity of a position function.

In this section, we will define the limit of a function, and look at numerical and graphical methods for computing them.

Ex) Let $f(x) = x^2 - 3x + 5$. What happens to $f$ as for values of $x$ near 3?

$x < 3$

| $x$ | $f$ |
|---|---|
| 2 | 3 |
| 2.5 | 3.75 |
| 2.9 | 4.71 |
| 2.99 | 4.9701 |
| 2.999 | 4.997 |
| 2.9999 | 4.9997 |

$x > 3$

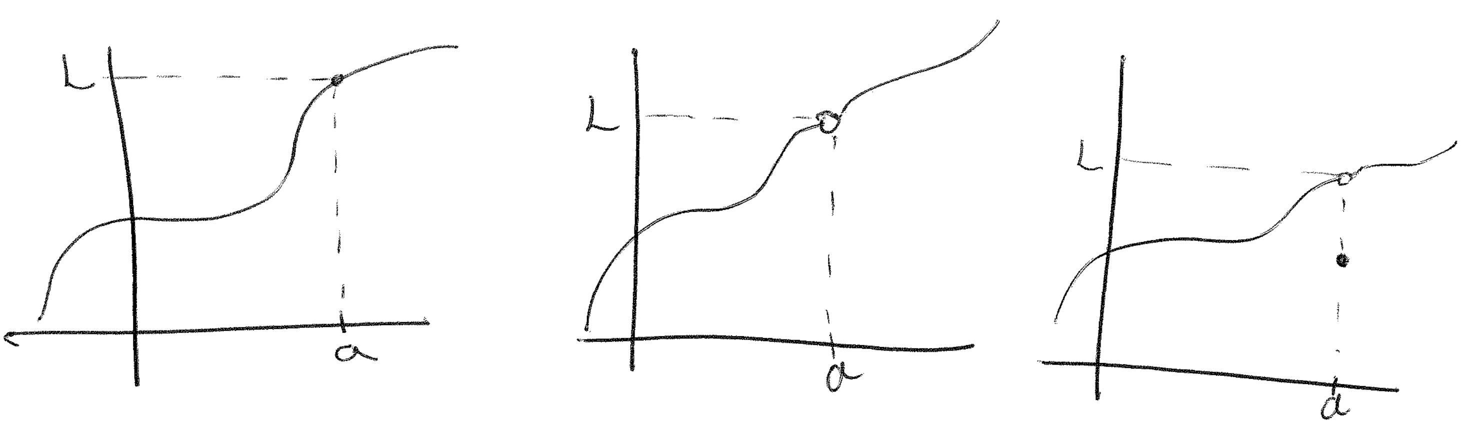
| $x$ | $f$ |
|---|---|
| 4 | 9 |
| 3.5 | 6.75 |
| 3.1 | 5.31 |
| 3.01 | 5.0301 |
| 3.001 | 5.003 |
| 3.0001 | 5.0003 |

as $x \to 3$
$f \to 5$

**Def** We say "the limit of $f(x)$ as $x$ approaches $a$ equals $L$" and we write $\lim\limits_{x\to a} f(x) = L$ or $f(x) \to L$ as $x \to a$ if we can make the values of $f(x)$ as close to $L$ as we like by taking $x$ sufficiently close to $a$ (on either side of $a$) **but not equal to $a$.**

***Why $x \neq a$?

three graphs



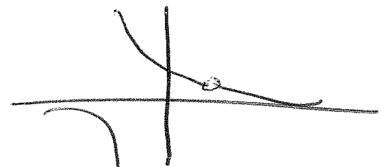
Ex) Guess the value of $\lim_{x \to 2} \frac{x-2}{x^2-4}$

note that $X \neq 2$ since $\frac{2-2}{2^2-4} = \frac{0}{0}$ undefined

| $X$ | $\frac{X-2}{X^2-4}$ |
|---|---|
| 1 | .333 |
| 1.5 | .28571 |
| 1.9 | .25641 |
| 1.99 | .25063 |

| $X$ | $\frac{X-2}{X^2-4}$ |
|---|---|
| 3 | 2 |
| 2.5 | .222 |
| 2.1 | .2439 |
| 2.01 | .24938 |

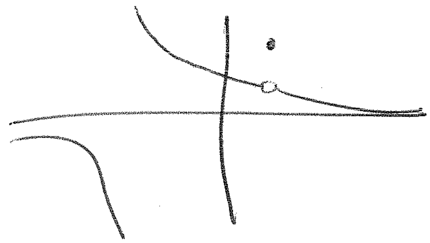
$\lim_{X \to 2} \frac{X-2}{X^2-4} = 1/4$



show table on calc.

Ex) Now let $g(x) = \begin{cases} \frac{x-2}{x^2-4} & if \ x \neq 2 \\ \pi \quad if & x = 2 \end{cases}$ Guess the value of $\lim_{x \to 2} g(x)$

$\lim_{X \to 2} g(x) = 1/4$



even though $g(2) = \pi$

It seems plausible that we should be able to find the value of a limit by simply "plugging in" values into the function, or by looking at a graph. However, pitfalls do arise. Let's take a look at examples 2,3,4 starting on page 102.

*We will learn more "foolproof" ways to evaluate limits in the coming sections.*

ex2. $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

| $t$ | $\frac{\sqrt{t^2+9}-3}{t^2}$ |
|---|---|
| 1 | .16228 |
| .5 | .16553 |
| .1 | .16662 |
| .01 | .16667 |

guess $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2} = 1/6$

but

| $t$ | $\frac{\sqrt{t^2+9}-3}{t^2}$ |
|---|---|
| .001 | .16667 |
| .0001 | .16667 |
| .00001 | .167 |
| .000001 | .2 |

So is it .16 or .2??
error in calculators rounding!

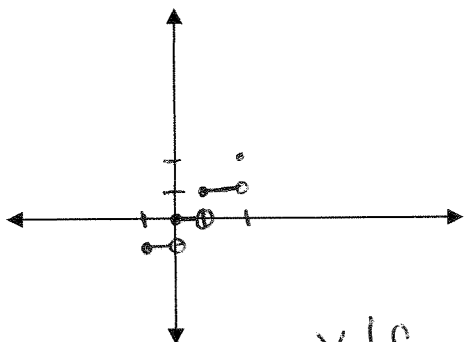
2.2

**One Sided Limits**

Consider $f(x) = \text{int}\,x$, the Greatest Integer Function, defined as the greatest integer less than or equal to $x$.

$f(x) = \lfloor x \rfloor$

Sketch $f(x) = \text{int}\,x$ on the interval [-1, 2].



Can we find $\lim_{x\to 1} \text{int}(x)$?

$x<1$

| x | f |
|---|---|
| 0 | 0 |
| .5 | 0 |
| .9 | 0 |
| .99 | 0 |

| x | f |
|---|---|
| 2 | 2 |
| 1.5 | 1 |
| 1.1 | 1 |
| 1.01 | 1 |

$x>1$

So is $\lim_{x\to 1} \text{int}(x) = 0$ or $1$

Well $\lim_{x\to 1^-} \text{int}(x) = 0$ and $\lim_{x\to 1^+} \text{int}(x) = 1$

*****In general, when does** $\lim_{x\to a} f(x)$ **exist?**

$\lim_{x\to a} f(x) = L$ iff $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$

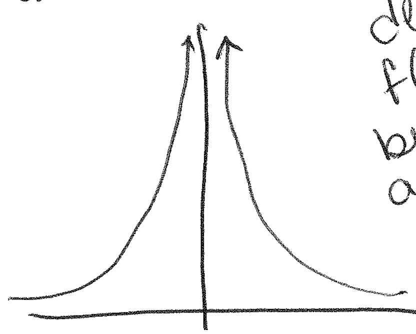
Ex) Let's look at #4 on page 106.

a. 3
b. 4
c. 2
d. does not exist
e. 3

dne

Ex) Find $\lim_{x\to 0} \frac{1}{x^2}$, if it exists.

| x | $1/x^2$ |
|---|---|
| ±1 | 1 |
| ±.5 | 4 |
| ±.1 | 100 |
| ±.01 | 10,000 |



def. of limit "we can make f(x) as close to L as we want" but f(x) isn't approaching any # it's just growing so

$\lim_{x\to 0} \frac{1}{x^2}$ dne.