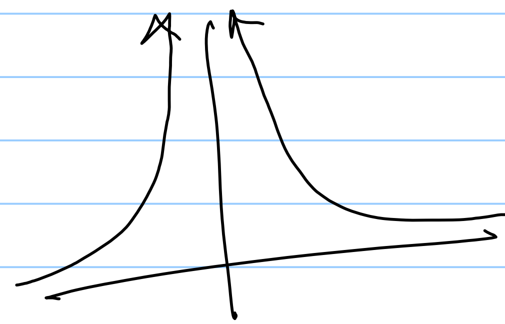
Note Title 8/7/2007

2.5

Infinite Limits

ex. note $\lim_{x\to 0} \frac{1}{x^2}$ d.n.e.

x	$\frac{1}{x^2}$
± 1	1
$\pm .5$	4
$\pm .1$	100
$\pm .01$	10,000
$\pm .001$	1,000,000



this implies $\lim_{x\to 0} \frac{1}{x^2} = \infty$

Definition

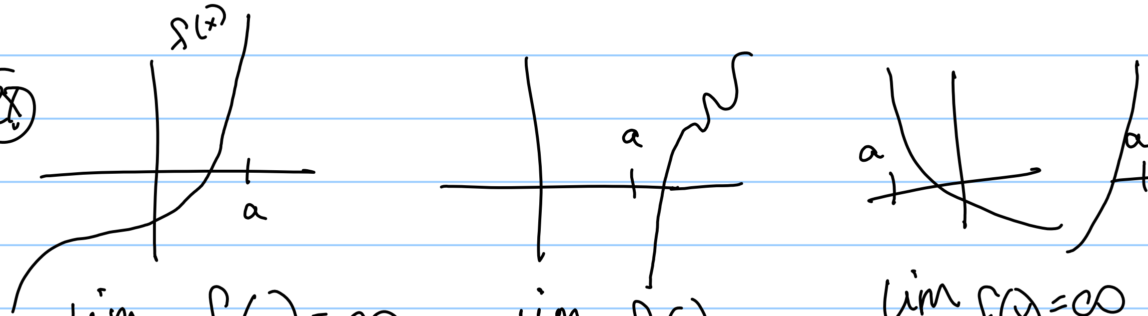
$\lim_{x\to a} f(x) = \infty$ means that $f(x)$ can be made as big as we want by making x close enough to a.

note: this does not imply that ∞ is a number
this is simply a notation

ex. 

$\lim_{x\to 2} g(x) = -\infty$

ex



$\lim_{x \to a^-} f(x) = \infty$ $\lim_{x \to a^+} f(x) = -\infty$ $\lim_{x \to a^+} f(x) = \infty$ $\lim_{x \to a^-} f(x) = \infty$

Defn.

The line $x=a$ is a vertical asymptote if at least one is true

$\lim_{x \to a} f(x) = \infty$ $\lim_{x \to a^-} f(x) = \infty$ $\lim_{x \to a^+} f(x) = \infty$

$\lim_{x \to a} f(x) = -\infty$ $\lim_{x \to a^-} f(x) = -\infty$ $\lim_{x \to a^+} f(x) = -\infty$

ex. the line $x=0$ is a VA of $f(x) = 1/x^2$ because $\lim_{x \to 0} f(x) = \infty$

ex. find $\lim_{x \to 5^-} \frac{3x}{x-5}$ and $\lim_{x \to 5^+} \frac{3x}{x-5}$

from left $\Rightarrow x < 5$

numerator is close to $3*5=15$

denominator is very small & negative

So $\approx \frac{15}{\text{small neg number}} \Rightarrow \lim_{x \to 5^-} \frac{3x}{x-5} = -\infty$

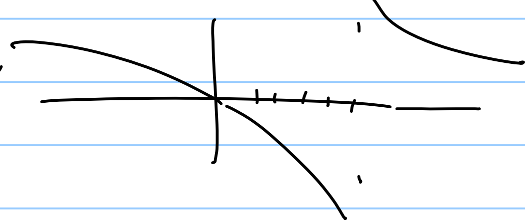
from right $\Rightarrow x > 5$

numerator is close to 15

denominator is very small & positive

So $\approx \frac{15}{\text{small pos number}} \Rightarrow \lim_{x \to 5^+} \frac{3x}{x-5} = \infty$

graph

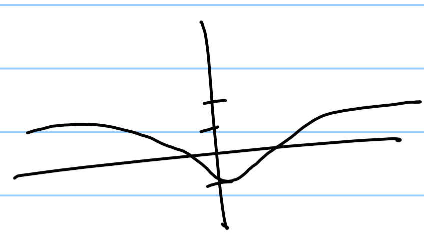


Limits at Infinity

ex. $f(x) = \dfrac{4x^2 - 1}{2x^2 + 1}$

x	$f(x)$
1	1
10	1.9851
100	1.9999

we write $\lim\limits_{x \to \infty} \dfrac{4x^2-1}{2x^2+1} = 2$



Defn

Let f be a function defined on (a, ∞)

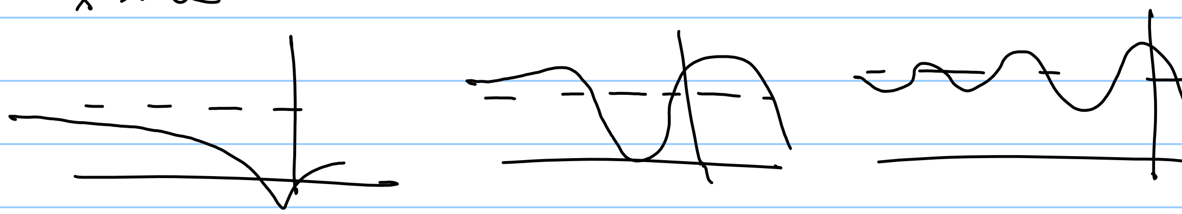
$\lim\limits_{x \to \infty} f(x) = L$

means that $f(x)$ can be as close to L as we want by making x large enough.

ex many ways

Also $\lim\limits_{x \to -\infty} f(x) = L$

ex.



<u>Defn.</u>

The line $y = L$ is a <u>horizontal asymptote</u> if either

$$\lim_{x \to \infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

Some limits at infinity that we know

$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2} \qquad \lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

$\lim\limits_{x \to \pm\infty} \frac{1}{x^n} = 0$ when n is positive integer

$\lim\limits_{x \to -\infty} a^x = 0$ when $a > 1$

ex. find $\lim\limits_{x \to 0^-} 5^{1/x}$

Let $t = 1/x$ then $\lim\limits_{x \to 0^-} t = -\infty$

$\lim\limits_{x \to 0^-} 5^{1/x} = \lim\limits_{t \to -\infty} 5^t = 0$ by above.

$\blacksquare$

Remember that ∞ is NOT A NUMBER!
So $\infty - \infty \neq 0$ $\frac{\infty}{\infty} \neq 1$ and so on.

ex) find $\lim_{x\to\infty} \frac{2x^2+5x-3}{3x^2-7x+10}$ need to do some algebra first

note $x \neq 0$ because $x \to \infty$

$$\lim_{x\to\infty} \frac{2x^2+5x-3}{3x^2-7x+10} \cdot \frac{1/x^2}{1/x^2}$$

$$= \lim_{x\to\infty} \frac{\frac{2x^2+5x-3}{x^2}}{\frac{3x^2-7x+10}{x^2}} = \lim_{x\to\infty} \frac{2+\frac{5}{x}-\frac{3}{x^2}}{3-\frac{7}{x}+\frac{10}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{5}{x} - \lim_{x\to\infty} \frac{3}{x^2}}{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \frac{7}{x} + \lim_{x\to\infty} \frac{10}{x^2}}$$

$$= \frac{2+0-0}{3-0+0} = \frac{2}{3}$$