2.6

Note Title 8/7/2007

Tangents

The tangent line to the curve $y=f(x)$ at the point $P(a,f(a))$ is the line through P w/ slope

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

if the limit exists.

Ex. Find an equation of the tangent line to $y=x^2$ at $P(1,1)$

$$m=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{x^2-1^2}{x-1}$$

$$=\lim_{x\to 1}\frac{x^2-1}{x-1}$$

$$=\lim_{x\to 1}\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}=\lim_{x\to 1} x+1$$

$$=1+1=2$$

so $m=2$ and through point $P(1,1)$

so $y-1=2(x-1)$

We often say, the slope of $y=x^2$ at $x=1$ is 2.

↳ idea: if we zoom in far enough the tangent line will look just like function.

There is an easier way to do this

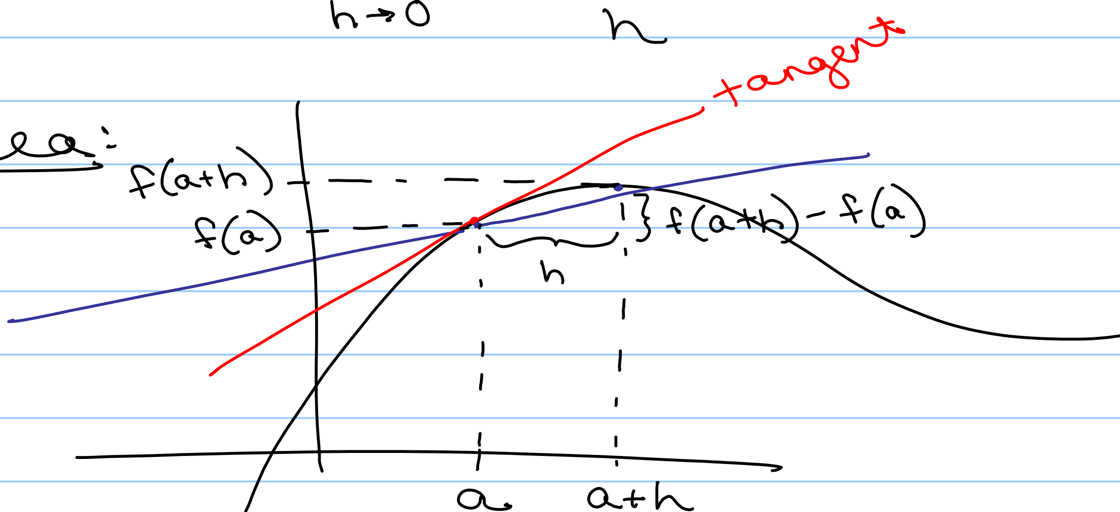
Let $h = x - a$

then $x = a + h$

$$\Rightarrow m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

idea:



$$m = \frac{f(a+h) - f(a)}{a + h - a} = \frac{f(a+h) - f(a)}{h}$$

to get the tangent line we want to make h very small, that is $h \to 0$

ex. Find equation of tangent line of $f(x) = 3x^2$ at $P(2, 12)$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3(2+h)^2 - 12}{h}$$

$$= \lim_{h \to 0} \frac{3(4+2h+h^2) - 12}{h}$$

$$= \lim_{h \to 0} \frac{12+6h+3h^2 - 12}{h}$$

$$= \lim_{h \to 0} \frac{6h+3h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(6+3h)}{h}$$

$$= \lim_{h \to 0} 6+3h$$

$$= 6 \quad \therefore m = 6$$

$$y - 12 = 6(x-2)$$

If $f(t)$ defines an objects position at time t (called a "position function")

The object moves from time a to time $a+h$ it's positive moves from $f(a)$ to $f(a+h)$

It's

$$\text{average velocity} = \frac{\text{distance moved}}{\text{time}} = \frac{f(a+h)-f(a)}{a+h-a}$$

to find the average velocity at an exact moment we find

$$v(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

called instantaneous velocity

In words the instantaneous velocity is equal to the slope of the tangent line at that point in time

ex. A dust bunny is moving across the floor by $s = 5t^2 + 8t$. Find velocity at $t=1$.

$$v(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{5(1+h)^2+8(1+h)-13}{h}$$

$$= \lim_{h\to 0} \frac{5(1+2h+h^2)+8+8h-13}{h}$$

$$= \lim_{h\to 0} \frac{5+10h+5h^2-5+8h}{h} = \lim_{h\to 0} \frac{18h+5h^2}{h}$$

$$= \lim_{h\to 0} 18+5h = 18 \text{ cm/sec}$$

Of course, it's doesn't have to be distance. It could be any change,...

temperature
height
weight
population growth
cost
area
volume
⋮