2.7

Note Title 8/6/2007

We saw that $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ was the slope of the tangent line and the instantaneous velocity. In fact this limit has so many uses that we have a special name/notation for it.

defn:

The <u>derivative of a function f at a number a</u>, denoted $f'(a)$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

ex. find the derivative of $f(x) = 1/x$ at $x = 4$

$$f'(4) = \lim_{h\to 0} \frac{\frac{1}{4+h} - \frac{1}{4}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{4}{4(4+h)} - \frac{4+h}{4(4+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{4-4-h}{4(4+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{4(4+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{4(4+h)} = \frac{-1}{4(4+0)} = \frac{-1}{16}$$

The tangent line to $y=f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$.

ex. Find an equation for tangent line of $f(x) = 1/x$ at $x=4$

from above we know $m = f'(4) = -\frac{1}{16}$

and $f(4) = 1/4$ so

$$y - 1/4 = -1/16(x-4)$$

Derivatives as Rates of Change

The derivative $f'(a)$ is the instaneous rate of change of $y=f(x)$ with respect to x when $x=a$.

The speed is the absolute value of the velocity.

ex. A population of bacteria, in minutes, is growing by $f(t)=2t^2+1$ find instantaneous growth rate at $t=2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{2(2+h)^2+1-9}{h}$$

$$= \lim_{h\to 0} \frac{2(4+4h+h^2)-8}{h}$$

$$= \lim_{h\to 0} \frac{8+8h+2h^2-8}{h}$$

$$= \lim_{h\to 0} 8+2h$$

$$= 8 \text{ bacteria/min}$$

So after exactly 2 minutes, 8 new bacteria are growing every minute