2.8

Note Title 8/7/2007

Derivative as a function

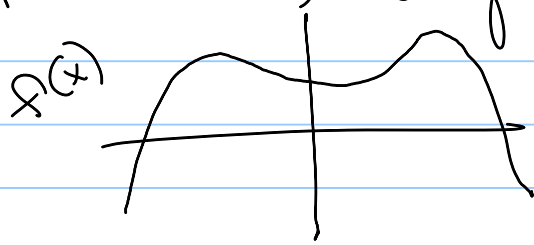
We talked about the derivative at a specific point a. But what if we want a to be a lot of different #s, say a variable x.

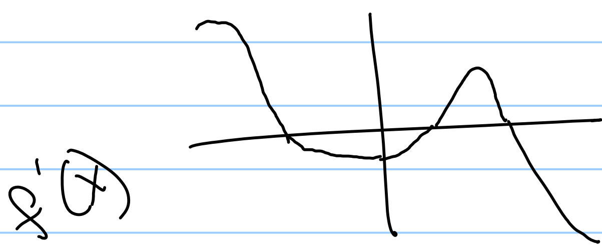
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

this is a function (vertical line test thing) it is called the derivative of f.

Remembering that $f'(x)$ can be interpreted as the slope of $f(x)$ we can use the graph of $f(x)$ to graph $f'(x)$

ex.





we can also use this defn to find a formula for $f'(x)$.

ex) find $f'(x)$ when $f(x) = 3x^2 + 4x + 7$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 + 4(x+h) + 7 - 3x^2 - 4x - 7}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 3x^2 - 4x}{h}$$

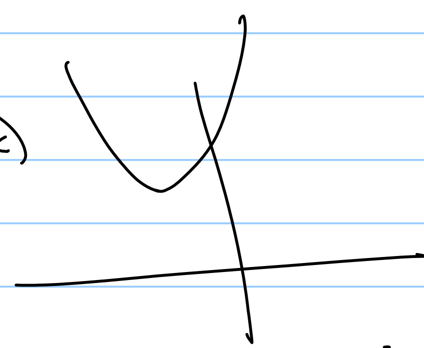
$$= \lim_{h \to 0} \frac{6xh + 3h^2 + 4h}{h}$$

$$= \lim_{h \to 0} 6x + 3h + 4$$

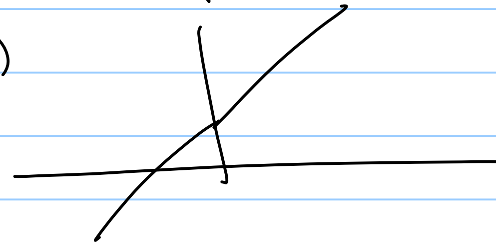
$$= 6x + 4$$

So $f'(x) = 6x + 4$

compare graphs $f(x)$



$f'(x)$



ex) find $f'(x)$ for $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

notice that domain of $f(x)$ is $\{x \mid x \geq 0\}$ but domain of $f'(x)$ is $\{x \mid x > 0\}$

Domains are not always the same (domain of $f'(x)$ may be smaller)

Other notations.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

also

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} = \left.\frac{dy}{dx}\right]_{x=a}$$

Defn

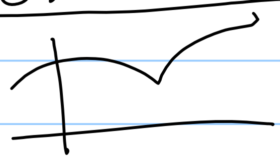
A function f is differentiable at a if $f'(a)$ exists

It is differentiable on an open interval (a,b) if it is differentiable at every number in the interval (or (a,∞) or $(-\infty,a)$ or $(-\infty,\infty)$)

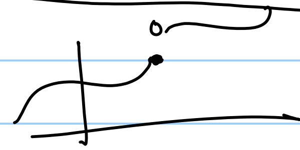
Thm

If f is differentiable at a, then f is continuous at a.

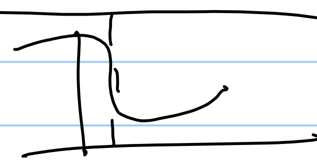
How can a function NOT be differentiable



corner



discontinuity



vertical tangent

Higher Derivatives

The second derivative, f'' (or $\frac{d^2y}{dx^2}$) is the derivative of the derivative of $f(x)$.

ex) find f'' when $f(x) = x^3+1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{+1}\cancel{-x^3}\cancel{-1}}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2 = 3x^2$$

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2hx+h^2)-3x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2}+6hx+3h^2\cancel{-3x^2}}{h}$$

$$= \lim_{h\to 0} 6x+3h = 6x$$

Note that the second derivative is
the rate of change of the rate of change
more commonly called <u>acceleration</u>

So if $s(t)$ is the position at time t
then $a(t) = s''(t)$ or $a(t) = v'(t)$
(where $a(t)$ is acceleration &
$v(t)$ is velocity)

The third derivative, f''', is the
derivative of f''

The 4^{th} derivative, $f^{(4)}$, is derivative of f'''
In general $f^{(n)}$ can be found by taking
the derivative of $f(x)$ n times.

In terms of $s(t)$, $v(t)$, $a(t)$,
$s'''(t) = j(t)$ called <u>jerk</u>

a large jerk $\Rightarrow$ a sudden change in $a(t)$