3.1

Note Title 8/16/2007

Derivatives of Polynomials & Exponentials

Derivative of a constant

$$\frac{d}{dx}(c) = 0$$

Proof Let $f(x) = c$ where c is a constant

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{c - c}{h}$$

$$= \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0$$

Power functions

$$\frac{d}{dx}(x) = 1$$

ex $\frac{d}{dx}(x^2) = \lim_{h\to 0} \frac{x^2+2hx+h^2-x^2}{h} = \lim_{h\to 0} \frac{2hx+h^2}{h}$

$$= \lim_{h\to 0} 2x+h = 2x$$

ex. $\frac{d}{dx}(x^3) = 3x^2$

$$\frac{d}{dx}(x^4) = 4x^3$$

The Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

ex. $\frac{d}{dx}(x^{17}) = 17x^{16}$

$\frac{d}{dx}(x^{-10}) = -10x^{-11}$

$\frac{d}{dx}\left(\frac{1}{x^4}\right) = \frac{d}{dx}(x^{-4}) = -4x^{-5} = \frac{-4}{x^5}$

$\frac{d}{dx}(\sqrt[4]{x}) = \frac{d}{dx}(x^{1/4}) = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$

ex. find an eqn. of tangent line to $y = x^3 + x^2$ at $(1,2)$

Slope of tangent is $f'(1)$

$f'(x) = 3x^2 + 2x$

$f'(1) = 3(1)^2 + 2(1)$

$= 3 + 2 = 5$

$y - 2 = 5(x-1)$

$$y-1=5(x-2)$$
$$y=5x-9$$

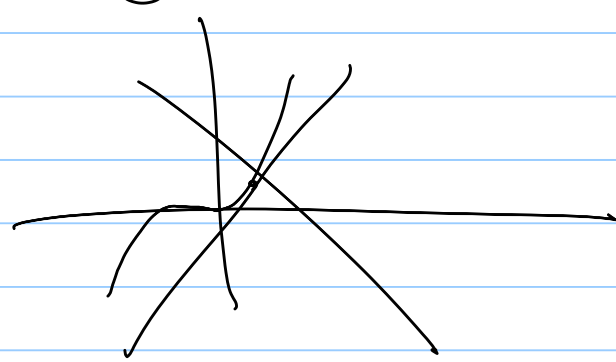
Normal line to a curve at point P is the line through P that is perpendicular to the tangent line.

ex) find normal line to $y=x^3+x^2$ at $(1,2)$

since $f'(1)=5$ we know the slope of normal line is $1/5$

so

$$y-1=-1/5(x-2)$$



News Derivatives from Old

If c is constant & f is differentiable

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}(f(x))$$

ex. $\frac{d}{dx}(5x^3) = 5\frac{d}{dx}(x^3) = 5 * 3x^2$
$= 15x^2$

$\frac{d}{dx}[-10x^7] = -70x^6$

Sum/Difference Rules

if f & g are differentiable

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

ex. $\frac{d}{dx}(5x^8 + 3x^6 - 8x^3 + x - 17)$

$= 40x^7 + 18x^5 - 24x^2 + 1$

s ⇒ meters
t ⇒ seconds

(ex) An equation of motion is
$$s(t) = 3t^5 - t^4 + 2t^2 + 8$$
find an equation for acceleration at time t
$$v(t) = s'(t) = 15t^4 - 4t^3 + 4t$$
$$a(t) = v'(t) = s''(t) = 60t^3 - 12t^2 + 4$$
whats acceleration at $t = 1$
$$a(1) = 60(1)^3 - 12(1)^2 + 4$$
$$= 60 - 12 + 4$$
$$= 52 \ m/s^2$$

Exponential Functions

Try to find derivative of $f(x) = a^x$
$$\frac{d}{dx}(a^x) = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x(a^h - 1)}{h}$$
$$= a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$
note $f'(0) = a^0 \lim_{h\to 0} \frac{a^h - 1}{h} = \lim_{h\to 0} \frac{a^h - 1}{h}$

so $f'(x) = a^x f'(0)$ or $\frac{d}{dx}(a^x) = a^x f'(0)$

It is helpful to have $f'(0) = 1$ because then we would have
$$\frac{d}{dx}(a^x) = a^x * 1 \text{ or just } \frac{d}{dx}(a^x) = a^x$$

so what # a gives $f'(0) = 1$

turns out to be a special #, e
the natural exponent!

Derivative of e^x

$$\frac{d}{dx}(e^x) = e^x$$

ex $f(x) = e^x + 8x^2$. find $f'(x)$

$f'(x) = e^x + 16x$

$f''(x) = e^x + 16$