3.5

Note Title 8/21/2007

Chain Rule

If g is differentiable at x & f is differentiable at $g(x)$ then $F = f \circ g$ (or written $f(g(x))$) is differentiable at x and is

$$F' = f'(g(x)) \cdot g'(x)$$

or $$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

ex. Find $F'(x)$ when $F(x) = \sqrt{x^2+8}$

if we let $f(x) = \sqrt{x}$ and $g(x) = x^2+8$

then $F(x) = f(g(x))$ and

$f'(x) = \frac{1}{2\sqrt{x}}$ and $g'(x) = 2x$

so

$$F'(x) = \frac{1}{2\sqrt{x^2+8}} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+8}}$$

Idea: derivative of the whole thing times the derivative of what's inside.

ex. find $f'(x)$ when $f(x) = \sin(x^3)$

$$f'(x) = \cos(x^3) \cdot 3x^2$$
$$= 3x^2 \cos x^3$$

ex. $g(x) = \sin^2(x) \longrightarrow (\sin x)^2$

$$g'(x) = 2\sin x \cdot \cos x$$

ex. $h(x) = (x^4 + 5x^2 + 7)^8$

$$h'(x) = 8(x^4 + 5x^2 + 7)^7 \cdot (4x^3 + 10x)$$

ex. $f(x) = (x^2+1)^4 (2x^3+6x)^6$

$$f'(x) = (x^2+1)^4 \, 6(2x^3+6x)^5 (6x^2+6) + (2x^3+6x)^6 \, 4(x^2+1)^3 (2x)$$

first   product rule   der. of 2nd

ex. $\sin(\cos(x^2))$ Chain Rule twice

$$\cos(\cos(x^2)) \frac{d}{dx} \cos(x^2)$$
$$\cos(\cos(x^2)) * -\sin(x^2)\, 2x$$
$$-2x \cos(\cos(x^2)) \sin(x^2)$$

Derivative of exponential functions.

$$\frac{d}{dx}(a^x) = a^x \ln a$$

for $a > 0$

Proof: find $\frac{d}{dx} a^x$ where $a > 0$

note: $e^{\ln a} = a$

$$\frac{d}{dx} a^x = \frac{d}{dx} (e^{\ln a})^x$$
$$= \frac{d}{dx} e^{(\ln a)x}$$
$$= e^{(\ln a)x} \frac{d}{dx}(\ln a)x$$
$$= e^{(\ln a)x} \ln a \frac{d}{dx} x$$
$$= e^{(\ln a)x} \ln a \cdot 1$$
$$= a^x \ln a$$

Tangents to Parametric Curves.

Let

$$x = f(t) \qquad y = g(t)$$ if everything is differentiable

then

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Ex. Find an equation of the tangent

$$x = \cos\theta + \sin 2\theta, \quad y = \sin\theta + \cos 2\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\frac{dy}{d\theta} = \cos\theta - 2\sin 2\theta \qquad \frac{dx}{d\theta} = -\sin\theta + 2\cos 2\theta$$

$$\frac{dy}{dx} = \frac{\cos\theta - 2\sin 2\theta}{-\sin\theta + 2\cos 2\theta}$$