3.6

Note Title 8/22/2007

Implicit Differentiation

Used when "y" is NOT a function of "x"

Approach
differentiate w/ respect to "x" using the chain Rule for "y".

ex. find y' (or $\frac{dy}{dx}$) for $x^5+y^2=9$

$$5x^4+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-5x^4}{2y}$$

ex. find y' for $x^2y^3+y^2\cos 2x=x$

$$x^2\,3y^2y'+y^3\,2x+y^2\cdot -\sin 2x\cdot 2+\cos 2x\cdot 2yy'=1$$

$$3x^2y^2y'+2xy^3-2y^2\sin 2x+2yy'\cos 2x=1$$

$$3x^2y^2y'+2yy'\cos 2x=1-2xy^3+2y^2\sin 2x$$

$$y'(3x^2y^2+2y\cos 2x)=1-2xy^3+2y^2\sin 2x$$

$$y'=\frac{1-2xy^3+2y^2\sin 2x}{3x^2y^2+2y\cos 2x}$$

Derivatives of Inverse Trig Functions

Reminder,

$y = \cos^{-1} x$ means $x = \cos y$

ex) $\cos 0 = 1$ so $\cos^{-1} 1 = 0$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$